

Holographic flavor on the Higgs branch

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ABSTRACT: In this paper we study the holographic dual, in several spacetime dimensions, of the Higgs branch of gauge theories with fundamental matter. These theories contain defects of various codimensionalities, where the matter fields are located. In the holographic description the matter is added by considering flavor brane probes in the supergravity backgrounds generated by color branes, while the Higgs branch is obtained when the color and flavor branes recombine with each other. We show that, generically, the holographic dual of the Higgs phase is realized by means of the addition of extra flux on the flavor branes and by choosing their appropriate embedding in the background geometry. This suggests a dielectric interpretation in terms of the color branes, whose vacuum solutions precisely match the F- and D-flatness conditions obtained on the field theory side. We further compute the meson mass spectra in several cases and show that when the defect added has codimension greater than zero it becomes continuous and gapless.

KEYWORDS: AdS-CFT Correspondence, Gauge-gravity correspondence, D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction

The gauge/gravity correspondence [1, 2] has been a breakthrough in our understanding of both gravity (and string theory) and gauge field theories. However, a major issue remains to be the inclusion of open string degrees of freedom, which would correspond to matter in the fundamental representation in the gauge field theory side of the correspondence.

A first step was taken in [3, 4], where it was suggested that one can add dynamical open string degrees of freedom by adding N_f intersecting Dq-branes to the original Dp-branes. In the limit in which the number of Dq-branes is much smaller than the number of Dp-branes, we can treat the system effectively as N_f probe branes in the background generated by the N_c Dp-branes, which, once we take the decoupling limit, will reduce to the corresponding near horizon geometry. Generically, the two types of branes overlap partially, which implies that the additional Dq-branes create a defect on the worldvolume theory of the Dp-branes. In the dual gauge theory description the extra branes give rise to additional matter, confined to live inside the defect, which comes from the Dp-Dq strings. When $q > p$, the decoupling limit forces the $SU(N_f)$ gauge symmetry on the Dq-brane to decouple. It then appears as a global flavor symmetry for the extra matter, which is in the fundamental representation of the flavor group. In this context, the fluctuations of the flavor branes should correspond to the mesons in the dual gauge theory. The study of these mesons was started in [5] for the D7-brane in the $AdS_5 \times S^5$ geometry, and it was further extended to other flavor branes in several backgrounds ([6]- [22]) (for a review see [23]).

The dual theories to these brane intersections are in the Coulomb phase. However one could have more involved situations, such as Higgs phases. On the field theory side the Higgs phase corresponds to having a non-zero VEV of the quark fields. As it is well-known (see e.g. ref. [24]) the Higgs branch of gauge theories can be realized in string theory by recombining color and flavor branes. This recombination can be described in two different and complementary ways. From the point of view of the flavor brane (the so-called macroscopic picture) the recombination is achieved by a non-trivial embedding of the brane probe in the background geometry and/or by a non-trivial flux of the worldvolume gauge field. On the other hand, the description of the recombination from the point of view of the color brane defines the microscopic picture. In most of the cases this microscopic picture can be regarded as a dielectric effect [25], in which a set of color branes gets polarized into a higher-dimensional flavor brane. Interestingly, the microscopic description of the Higgs branch allows a direct relation with the field theory analysis and the micro-macro matching is essential to understand how gauge theory quantities are encoded into the configuration of the flavor brane.

In ref. [26] the Higgs phase of the D3-D7 intersection was studied (see also ref. [27]). It was proposed in [26, 27] that, from the point of view of the D7-brane, one can realize a (mixed Coulomb-)Higgs phase of the D3-D7 system by switching on an instanton configuration of the worldvolume gauge field of the D7-brane. This instantonic gauge field lives on the directions of the D7-brane worldvolume that are orthogonal to the gauge theory directions. The size of the instanton has been identified in [26, 27] with the VEV of the quark fields. The meson spectra depends on this size and was shown in [26] to display, in the limit of infinite instanton size, an spectral flow phenomenon.

The defect conformal field theory associated to the D3-D3 intersection was studied in ref. [28], where the corresponding fluctuation/operator dictionary was established. The meson mass spectra of this system when the two sets of D3-branes are separated was computed analytically in ref. [20]. In [28] the Higgs branch of the D3-D3 system was identified as a particular holomorphic embedding of the probe D3-brane in the $AdS_5 \times S^5$

geometry, which was shown to correspond to the vanishing of the F- and D-terms in the dual superconformal field theory (see also refs. [29, 30]).

The Higgs phase of the dual to the D3-D5 intersection was discussed in ref. [31]. On the field theory side [32] the D3-D5 system describes the dynamics of a $2 + 1$ dimensional defect containing fundamental hypermultiplets living inside the $3 + 1$ dimensional $\mathcal{N} = 4$ SYM. The meson spectra on the Coulomb branch has been obtained in [20]. In [31] it was found that, in the supergravity dual, the Higgs phase corresponds to adding magnetic worldvolume flux inside the flavor D5-brane transverse to the D3-branes. This worldvolume gauge field has the non-trivial effect of inducing D3-brane charge in the D5-brane worldvolume, which in turn suggests an alternative microscopical description in terms of D3-branes expanded to a D5-brane due to dielectric effect [25]. Indeed, the vacuum conditions of the dielectric theory can be mapped to the F and D flatness constraints of the dual gauge theory, thus justifying the identification with the Higgs phase. The Higgs vacua of the field theory involve a non-trivial dependence of the defect fields on the coordinate transverse to the defect. In the supergravity side this is mapped to a bending of the flavor brane, which is actually required by supersymmetry (see [33]). Moreover, in [31] the spectrum of transverse fluctuations was computed in the Higgs phase, with the result that the discrete spectrum is lost. The reason is that the IR theory is modified because of the non-trivial profile of the flavor brane, so that in the Higgs phase, instead of having an effective $AdS \times S$ worldvolume for the flavor brane, one has Minkowski space, thus losing the KK-scale which would give rise to a discrete spectrum.

In this paper we will generalize the results on the Higgs branch of refs. [26, 28] and [31] to the rest of supersymmetric brane intersections. In general, each type of intersection is dual to a defect hosting a field theory living inside a bulk gauge theory. Therefore, we can label each case by the codimensionality of the defect. We will see that generically all of them behave in a similar way to the D3-D5 case, in the sense that the Higgs phase is achieved by adding extra worldvolume flux to the flavor brane. However, as we will see, in not all the cases the discrete meson spectrum is lost.

We begin in section 2 by analyzing the codimension zero defect, which corresponds to the Dp-D(p+4) intersection. We first study the field theory of the D3-D7 system, where we identify a mixed Coulomb-Higgs branch which is given by the vanishing of both the D- and F-terms. This branch is characterized by a non-zero commutator of the adjoint fields of $\mathcal{N} = 4$ SYM which, from the point of view of the flavor brane, corresponds to having a non-vanishing flux of the worldvolume gauge field along the directions orthogonal to the color brane. We will then describe such non-commutative scalars by using the Myers action for a dielectric D3-brane and we will argue that, macroscopically, this configuration can be described in terms of a D7-brane with a self-dual instantonic gauge field. From this matching between the D3- and D7-brane descriptions we will be able to extract the relation proposed in ref. [26] between the VEV of the quark field and the size of the instanton. Afterwards we perform the computation of the meson spectrum of the general Dp-D(p+4) systems, which in this case remains discrete. We estimate the value of the mesonic mass gap as a function of the instanton size. For large instantons this gap is independent of the size, in agreement with the spectral flow found in ref. [26], while for small instantons the

mass gap is proportional to the size of the instanton and vanishes in the zero-size limit.

In section 3 we discuss the codimension one defects, whose most prominent example is the D3-D5 intersection studied in [31]. In this paper we study the general Dp-D(p+2) case with worldvolume flux on the D(p+2)-brane, which also admits a dielectric interpretation. We then analyze the meson spectrum, which is continuous and gapless as in the D3-D5 case studied in [31]. We will establish this result for the full set of fluctuations of the D(p+2)-brane probe, which are analyzed in appendix A by using the same techniques as those employed in refs. [20, 21] to study the Coulomb branch. We then study the F1-Dp intersection which, in particular, for $p = 3$ corresponds to the S-dual version of the D1-D3 system.

In section 4 we study a close relative to the Dp-D(p+2) intersection, namely the M2-M5 intersection in M-theory. In this case, we see that we can dissolve M2-branes by turning on a three-form flux on the M5-brane worldvolume and introducing some bending of the M5-brane. The supersymmetry of this configuration is explicitly confirmed in appendix B by looking at the kappa symmetry of the embedding. However, in this case a microscopical description is not at hand, since it would involve an action for coincident M2-branes which is not known at present.

Section 5 is devoted to the analysis of the codimension two defects, which correspond to the Dp-Dp intersections. This case, as anticipated in [28, 29], is somehow different, since the Higgs phase is realized by the choice of a particular embedding of the probe Dp-brane with no need of extra flux. This case is rather particular since, as we will show, the profile can be an arbitrary holomorphic curve in suitable coordinates, although only one of them gives the desired Higgs phase, while the rest remain unidentified.

We then finish in section 6 with some conclusions.

2. The codimension zero defect

Let us start considering the D3-D7 intersection, where the D3-branes are fully contained in the D7-branes as shown in the following array:

$$\begin{array}{cccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 D3 : & \times & \times & \times & - & - & - & - & - & - \\
 D7 : & \times & \times & \times & \times & \times & \times & \times & - & -
 \end{array} \tag{2.1}$$

Clearly, the D3-D7 string sector gives rise to extra fundamental matter living in the $3 + 1$ common directions. It can be seen that the dual gauge theory is a $\mathcal{N} = 2$ SYM theory in $3 + 1$ dimensions obtained by adding N_f $\mathcal{N} = 2$ fundamental hypermultiplets to the $\mathcal{N} = 4$ SYM theory. We can further break the classical conformal invariance of the theory by adding a mass term for the quark hypermultiplets. The lagrangian is given by ([26]):

$$\begin{aligned}
 \mathcal{L} = & \tau \int d^2\theta d^2\bar{\theta} \left(\text{tr}(\Phi_I^\dagger e^V \Phi_I e^{-V}) + Q_i^\dagger e^V Q^i + \tilde{Q}_i e^{-V} \tilde{Q}^{i\dagger} \right) \\
 & + \tau \int d^2\theta \left(\text{tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) + W \right) + \tau \int d^2\bar{\theta} \left(\text{tr}(\bar{\mathcal{W}}_{\dot{\alpha}} \bar{\mathcal{W}}^{\dot{\alpha}}) + \bar{W} \right), \tag{2.2}
 \end{aligned}$$

together with the equation:

$$Q^i \tilde{Q}_i + [\Phi_1, \Phi_2] = 0 . \tag{2.8}$$

In (2.8) $Q^i \tilde{Q}_i$ denotes a matrix in color space of components $Q_a^i \tilde{Q}_i^b$. For a vacuum election as in eq. (2.6) we can restrict ourselves to the lower $k \times k$ matrix block, and we can write eq. (2.8) as:

$$q^i \tilde{q}_i + [\Phi_1, \Phi_2] = 0 , \tag{2.9}$$

where now, and it what follows, it is understood that Φ_1 and Φ_2 are $k \times k$ matrices.

Eq. (2.9) contains an important piece of information since it shows that a non-vanishing VEV of the quark fields q and \tilde{q} induces a non-zero commutator of the adjoint fields Φ_1 and Φ_2 . Therefore, in the Higgs branch, some scalars transverse to the D3-brane are necessarily non-commutative. Notice that Φ_1 and Φ_2 correspond precisely to the directions transverse to the D3-brane which lie on the worldvolume of the D7-brane (i.e. they correspond to the directions 4, \dots 7 in the array (2.1)). This implies that the description of this intersection from the point of view of the D7-branes must involve a non-trivial configuration of the worldvolume gauge field components of the latter along the directions 4, \dots 7. We will argue in the next subsection that this configuration corresponds to switching on an instantonic flux along these directions.

In order to match the field theory vacuum with our brane description we should also be able to reproduce the D -flatness condition arising from the lagrangian (2.2). Assuming that the quark fields \tilde{Q} and Q are only non-vanishing on the lower $k \times k$ block, we can write this condition as:

$$|q^i|^2 - |\tilde{q}_i|^2 + [\Phi_1, \Phi_1^\dagger] + [\Phi_2, \Phi_2^\dagger] = 0 . \tag{2.10}$$

The constraints (2.9) and (2.10), together with the condition $[\Phi^I, \Phi^3] = 0$, define the mixed Coulomb-Higgs phase of the theory.

2.1 Gravity dual of the mixed Coulomb-Higgs phase

As it is well-known, there is a one-to-one correspondence between the Higgs phase of $\mathcal{N} = 2$ gauge theories and the moduli space of instantons ([34–36]). This comes from the fact that the F - and D -flatness conditions can be directly mapped into the ADHM equations (see [37] for a review). Because of this map, we can identify the Higgs phase of the gauge theory with the space of 4d instantons. In the context of string theory, a $\mathcal{N} = 2$ theory can be engineered by intersecting Dp with D(p+4) branes over a $p + 1$ dimensional space. In particular, if we consider the D3-D7 system, the low energy effective lagrangian is precisely given by (2.2). In this context, the Higgsing of the theory amounts to adding some units of instantonic DBI flux in the subspace transverse to the D3 but contained in the D7, which provides a natural interpretation of the Higgs phase-ADHM equations map.

Let us analyze this in more detail. Suppose we have N D3-branes and N_f D7-branes. In the field theory limit in which we take α' to zero but keeping fixed the Yang-Mills coupling of the theory on the D3's, the gauge dynamics on the D7-brane is decoupled. Then, the $SU(N_f)$ gauge symmetry of the D7-brane is promoted to a global $SU(N_f)$ flavor

symmetry on the effective theory describing the system, which is $\mathcal{N} = 4$ SYM plus N_f $\mathcal{N} = 2$ hypermultiplets arising from the D3-D7 strings; and whose lagrangian is the one written in (2.2). The gravity dual of this theory would be obtained by replacing the branes by their backreacted geometry and taking the appropriate low energy limit. However, in the limit in which $N_f \ll N$ we can consider the D7-branes as probes in the near-horizon geometry created by the D3-branes, namely $AdS_5 \times S^5$:

$$ds^2 = \frac{r^2}{R^2} dx_{1,3}^2 + \frac{R^2}{r^2} dr^2, \tag{2.11}$$

where \vec{r} is the six-dimensional vector along the directions orthogonal to the stack of D3-branes and the radius R is given by $R^4 = 4\pi g_s N (\alpha')^2$. In addition, $dx_{1,3}^2$ is the metric of the 3 + 1 dimensional Minkowski space along which the D3-branes lie. The type IIB supergravity background also includes a 4-form RR potential given by:

$$C^{(4)} = \left(\frac{r^2}{R^2} \right)^2 dx^0 \wedge \dots \wedge dx^3. \tag{2.12}$$

Let us now write the $AdS_5 \times S^5$ background in a system of coordinates more suitable for our purposes. Let $\vec{y} = (y^1, \dots, y^4)$ be the coordinates along the directions 4, \dots , 7 in the array (2.1) and let us denote by ρ the length of \vec{y} (i.e. $\rho^2 = \vec{y} \cdot \vec{y}$). Moreover, we will call $\vec{z} = (z^1, z^2)$ the coordinates 8, 9 of (2.1). Notice that \vec{z} is a vector in the directions which are orthogonal to both stacks of D-branes. Clearly, $r^2 = \rho^2 + \vec{z}^2$ and the metric (2.11) can be written as:

$$ds^2 = \frac{\rho^2 + \vec{z}^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2 + \vec{z}^2} (d\vec{y}^2 + d\vec{z}^2). \tag{2.13}$$

The Dirac-Born-Infeld (DBI) action for a stack of N_f D7-branes is given by:

$$S_{\text{DBI}}^{D7} = -T_7 \int d^8 \xi e^{-\phi} \text{Str} \left\{ \sqrt{-\det(g + F)} \right\}, \tag{2.14}$$

where ξ^a is a system of worldvolume coordinates, ϕ is the dilaton, g is the induced metric and F is the field strength of the $SU(N_f)$ worldvolume gauge group¹. Let us assume that we take $\xi^a = (x^\mu, y^i)$ as worldvolume coordinates and that we consider a D7-brane embedding in which $|\vec{z}| = L$, where L represents the constant transverse separation between the two stacks of D3- and D7- branes. Notice that this transverse separation will give a mass $L/2\pi\alpha'$ to the D3-D7 strings, which corresponds to the quark mass in the field theory dual. For an embedding with $|\vec{z}| = L$, the induced metric takes the form:

$$g_{x^\mu x^\nu} = \frac{\rho^2 + L^2}{R^2} \eta_{\mu\nu}, \quad g_{y^i y^j} = \frac{R^2}{\rho^2 + L^2} \delta_{ij}. \tag{2.15}$$

Let us now assume that the worldvolume field strength F has non-zero entries only along the directions of the y^i coordinates and let us denote $F_{y^i y^j}$ simply by F_{ij} . Then, after using

¹Notice that, with our notations, F_{ab} is dimensionless and, therefore, the relation between F_{ab} and the gauge potential A is $F_{ab} = \partial_a A_b - \partial_b A_a + \frac{1}{2\pi\alpha'} [A_a, A_b]$, whereas the gauge covariant derivative is $D_a = \partial_a + \frac{1}{2\pi\alpha'} A_a$.

eq. (2.15) and the fact that the dilaton is trivial for the $AdS_5 \times S^5$ background, the DBI action (2.14) takes the form:

$$S_{\text{DBI}}^{D7} = -T_7 \int d^4x d^4y \text{Str} \left\{ \sqrt{\det \left(\delta_{ij} + \left(\frac{\rho^2 + L^2}{R^2} \right) F_{ij} \right)} \right\}. \quad (2.16)$$

The matrix appearing on the right-hand side of eq. (2.16) is a 4×4 matrix whose entries are $SU(N_f)$ matrices. However, inside the symmetrized trace such matrices can be considered as commutative numbers. Actually, we will evaluate the determinant in (2.16) by means of the following identity. Let $M_{ij} = -M_{ji}$ be a 4×4 antisymmetric matrix. Then, one can check that:

$$\det(1 + M) = 1 + \frac{1}{2} M^2 + \frac{1}{16} (*M M)^2, \quad (2.17)$$

where M^2 and $*M M$ are defined as follows:

$$M^2 \equiv M_{ij} M_{ij}, \quad *M M \equiv *M_{ij} M_{ij}, \quad (2.18)$$

and $*M$ is defined as the following matrix:

$$*M_{ij} = \frac{1}{2} \epsilon_{ijkl} M_{kl}. \quad (2.19)$$

When the M_{ij} matrix is self-dual (i.e. when $*M = M$), the three terms on the right-hand side of (2.17) build up a perfect square. Indeed, one can check by inspection that, in this case, one has:

$$\det(1 + M) \Big|_{\text{self-dual}} = \left(1 + \frac{1}{4} M^2 \right)^2. \quad (2.20)$$

Let us apply these results to our problem. First of all, by using (2.17) one can rewrite eq. (2.16) as:

$$S_{\text{DBI}}^{D7} = -T_7 \int d^4x d^4y \text{Str} \left\{ \sqrt{1 + \frac{1}{2} \left(\frac{\rho^2 + L^2}{R^2} \right)^2 F^2 + \frac{1}{16} \left(\frac{\rho^2 + L^2}{R^2} \right)^4 (*FF)^2} \right\}. \quad (2.21)$$

Let us now consider the Wess-Zumino(WZ) piece of the worldvolume action. For a D7-brane in the $AdS_5 \times S^5$ background this action reduces to:

$$S_{\text{WZ}}^{D7} = \frac{T_7}{2} \int \text{Str} \left[P[C^{(4)}] \wedge F \wedge F \right], \quad (2.22)$$

where $P[\dots]$ denotes the pullback of the form inside the brackets to the worldvolume of the D7-brane. By using the same set of coordinates as in (2.16), and the explicit expression of $C^{(4)}$ (see eq. (2.12)), one can rewrite S_{WZ}^{D7} as:

$$S_{\text{WZ}}^{D7} = T_7 \int d^4x d^4y \text{Str} \left\{ \frac{1}{4} \left(\frac{\rho^2 + L^2}{R^2} \right)^2 *FF \right\}. \quad (2.23)$$

Let us now consider a configuration in which the worldvolume gauge field is self-dual in the internal \mathbb{R}^4 of the worldvolume spanned by the y^i coordinates which, as one can check, satisfies the equations of motion of the D7-brane probe. For such an instantonic gauge configuration $*F = F$, where $*F$ is defined as in eq. (2.19). As in eq. (2.20), when $F = *F$ the DBI action (2.21) contains the square root of a perfect square and we can write:

$$S_{\text{DBI}}^{D7}(\text{self - dual}) = -T_7 \int d^4x d^4y \text{Str} \left\{ 1 + \frac{1}{4} \left(\frac{\rho^2 + L^2}{R^2} \right)^2 *FF \right\}. \quad (2.24)$$

Moreover, by comparing eqs. (2.23) and (2.24) one readily realizes that the WZ action cancels against the second term of the right-hand side of eq. (2.24). To be more explicit, once we assume the instantonic character of F , the full action for a self-dual configuration is just:

$$S^{D7}(\text{self - dual}) = -T_7 \int d^4x d^4y \text{Str} [1] = -T_7 N_f \int d^4x d^4y. \quad (2.25)$$

Notice that in the total action (2.25) the transverse distance L does not appear. This “no-force” condition is an explicit manifestation of the SUSY of the system. Indeed, the fact that the DBI action is a square root of a perfect square is required for supersymmetry, and actually can be regarded as the saturation of a BPS bound.

In order to get a proper interpretation of the role of the instantonic gauge field on the D7-brane probe, let us recall that for self-dual configurations the integral of the Pontryagin density $\mathcal{P}(y)$ is quantized for topological reasons. Actually, with our present normalization of F , $\mathcal{P}(y)$ is given by:

$$\mathcal{P}(y) \equiv \frac{1}{16\pi^2} \frac{1}{(2\pi\alpha')^2} \text{tr} [*FF], \quad (2.26)$$

and, if $k \in \mathbb{Z}$ is the instanton number, one has:

$$\int d^4y \mathcal{P}(y) = k. \quad (2.27)$$

A worldvolume gauge field satisfying (2.27) is inducing k units of D3-brane charge into the D7-brane worldvolume along the subspace spanned by the Minkowski coordinates x^μ . To verify this fact, let us rewrite the WZ action (2.22) of the D7-brane as:

$$S_{\text{WZ}}^{D7} = \frac{T_7}{4} \int d^4x d^4y C_{x^0x^1x^2x^3}^{(4)} \text{tr} [*FF] = T_3 \int d^4x d^4y C_{x^0x^1x^2x^3}^{(4)} \mathcal{P}(y), \quad (2.28)$$

where we have used (2.26) and the relation $T_3 = (2\pi)^4 (\alpha')^2 T_7$ between the tensions of the D3- and D7-branes. If $C_{x^0x^1x^2x^3}^{(4)}$ does not depend on the coordinate y , we can integrate over y by using eq. (2.27), namely:

$$S_{\text{WZ}}^{D7} = k T_3 \int d^4x C_{x^0x^1x^2x^3}^{(4)}. \quad (2.29)$$

Eq. (2.29) shows that the coupling of the D7-brane with k instantons in the worldvolume to the RR potential $C^{(4)}$ of the background is identical to the one corresponding to k D3-branes, as claimed above. It is worth to remark here that the existence of these instanton configurations relies on the fact that we are considering $N_f > 1$ flavor D7 branes, i.e. that we have a non-abelian worldvolume gauge theory.

2.2 A microscopical interpretation of the D3-D7 intersection with flux

The fact that the D7-branes carry k dissolved D3-branes on them opens up the possibility of a new perspective on the system, which could be regarded not just from the point of view of the D7-branes with dissolved D3s, but also from the point of view of the dissolved D3-branes which expand due to dielectric effect [25] to a transverse fuzzy \mathbb{R}^4 . To see this, let us assume that we have a stack of k D3-branes in the background given by (2.13). These D3-branes are extended along the four Minkowski coordinates x^μ , whereas the transverse coordinates \vec{y} and \vec{z} must be regarded as the matrix scalar fields Y^i and Z^j , taking values in the adjoint representation of $SU(k)$. Actually, we will assume in what follows that the Z^j scalars are abelian, as it corresponds to a configuration in which the D3-branes are localized (i.e. not polarized) in the space transverse to the D7-brane.

The dynamics of a stack of coincident D3-branes is determined by the Myers dielectric action [25], which is the sum of a Dirac-Born-Infeld and a Wess-Zumino part:

$$S_{D3} = S_{\text{DBI}}^{D3} + S_{\text{WZ}}^{D3} . \tag{2.30}$$

For the background we are considering the Born-Infeld action is:

$$S_{\text{DBI}}^{D3} = -T_3 \int d^4\xi \mathbf{Str} \left[\sqrt{-\det \left[P[G + G(Q^{-1} - \delta)G]_{ab} \right]} \sqrt{\det Q} \right] . \tag{2.31}$$

In eq. (2.31) G is the background metric, $\mathbf{Str}(\dots)$ represents the symmetrized trace over the $SU(k)$ indices and Q is a matrix which depends on the commutator of the transverse scalars (see below). The Wess-Zumino term for the D3-brane in the $AdS_5 \times S^5$ background under consideration is:

$$S_{\text{WZ}}^{D3} = T_3 \int d^4\xi \mathbf{Str} \left[P[C^{(4)}] \right] . \tag{2.32}$$

As we are assuming that only the Y scalars are non-commutative, the only elements of the matrix Q appearing in (2.31) that differ from those of the unit matrix are given by:

$$Q_{y^i y^j} = \delta_{ij} + \frac{i}{2\pi\alpha'} [Y^i, Y^k] G_{y^k y^j} . \tag{2.33}$$

By using the explicit form of the metric elements along the y coordinates (see eq. (2.13)), one can rewrite Q_{ij} as:

$$Q_{y^i y^j} = \delta_{ij} + \frac{i}{2\pi\alpha'} \frac{R^2}{\hat{r}^2} [Y^i, Y^j] , \tag{2.34}$$

where \hat{r}^2 is the matrix:

$$\hat{r}^2 = (Y^i)^2 + Z^2 . \tag{2.35}$$

Let us now define the matrix θ_{ij} as:

$$i\theta_{ij} \equiv \frac{1}{2\pi\alpha'} [Y^i, Y^j] . \tag{2.36}$$

It follows from this definition that θ_{ij} is antisymmetric in the i, j indices and, as a matrix of $SU(k)$, is hermitian:

$$\theta_{ij} = -\theta_{ji} , \quad \theta_{ij}^\dagger = \theta_{ij} . \quad (2.37)$$

Moreover, in terms of θ_{ij} , the matrix Q_{ij} can be written as:

$$Q_{y^i y^j} = \delta_{ij} - \frac{R^2}{\hat{r}^2} \theta_{ij} . \quad (2.38)$$

Using these definitions, we can write the DBI action (2.31) for the dielectric D3-brane in the $AdS_5 \times S^5$ background as:

$$S_{\text{DBI}}^{D3} = -T_3 \int d^4x \text{Str} \left[\left(\frac{\hat{r}^2}{R^2} \right)^2 \sqrt{\det \left(\delta_{ij} - \frac{R^2}{\hat{r}^2} \theta_{ij} \right)} \right] , \quad (2.39)$$

where we have chosen the Minkowski coordinates x^μ as our set of worldvolume coordinates for the dielectric D3-brane. Similarly, the WZ term can be written as:

$$S_{\text{WZ}}^{D3} = T_3 \int d^4x \text{Str} \left[\left(\frac{\hat{r}^2}{R^2} \right)^2 \right] . \quad (2.40)$$

Let us now assume that the matrices θ_{ij} are self-dual with respect to the ij indices, i.e. that $*\theta = \theta$. Notice that, in terms of the original matrices Y^i , this is equivalent to the condition:

$$[Y^i, Y^j] = \frac{1}{2} \epsilon_{ijkl} [Y^k, Y^l] . \quad (2.41)$$

Moreover, the self-duality condition implies that there are three independent θ_{ij} matrices, namely:

$$\theta_{12} = \theta_{34} , \quad \theta_{13} = \theta_{42} , \quad \theta_{14} = \theta_{23} . \quad (2.42)$$

The description of the D3-D7 system from the perspective of the color D3-branes should match the field theory analysis performed at the beginning of this section. In particular, the D- and F-flatness conditions of the adjoint fields in the Coulomb-Higgs phase of the $\mathcal{N} = 2$ SYM with flavor should be the same as the ones satisfied by the transverse scalars of the dielectric D3-brane. In order to check this fact, let us define the following complex combinations of the Y^i matrices:

$$2\pi\alpha' \Phi_1 \equiv \frac{Y^1 + iY^2}{\sqrt{2}} , \quad 2\pi\alpha' \Phi_2 \equiv \frac{Y^3 + iY^4}{\sqrt{2}} , \quad (2.43)$$

where we have introduced the factor $2\pi\alpha'$ to take into account the standard relation between coordinates and scalar fields in string theory. We are going to identify Φ_1 and Φ_2 with the adjoint scalars of the field theory side. To verify this identification, let us compute the commutators of these matrices and, as it was done in [31], let us match them with the ones obtained from the F-flatness conditions of the field theory analysis. From the definitions (2.36) and (2.43) and the self-duality condition (2.42), it is straightforward to check that:

$$\begin{aligned} [\Phi_1, \Phi_2] &= -\frac{\theta_{23}}{2\pi\alpha'} + i\frac{\theta_{13}}{2\pi\alpha'} , \\ [\Phi_1, \Phi_1^\dagger] &= [\Phi_2, \Phi_2^\dagger] = \frac{\theta_{12}}{2\pi\alpha'} . \end{aligned} \quad (2.44)$$

By comparing with the results of the field theory analysis (eqs. (2.9) and (2.10)), we get the following identifications between the θ 's and the vacuum expectation values of the matter fields:

$$q^i \tilde{q}_i = \frac{\theta_{23}}{2\pi\alpha'} - i \frac{\theta_{13}}{2\pi\alpha'}, \quad |\tilde{q}_i|^2 - |q^i|^2 = \frac{\theta_{12}}{\pi\alpha'}. \quad (2.45)$$

Moreover, from the point of view of this dielectric description, the Φ_3 field in the field theory is proportional to $Z^1 + iZ^2$. Since the stack of branes is localized in that directions, Z^1 and Z^2 are abelian and clearly we have that $[\Phi_1, \Phi_3] = [\Phi_2, \Phi_3] = 0$, thus matching the last F -flatness condition for the adjoint field Φ_3 .

It is also interesting to relate the present “microscopic” description of the D3-D7 intersection, in terms of a stack of dielectric D3-branes, to the “macroscopic” description of subsection 2.1, in terms of the flavor D7-branes. With this purpose in mind, let us compare the actions of the D3- and D7-branes. First of all, we notice that, when the matrix θ is self-dual, we can use eq. (2.20) and write the DBI action (2.39) as:

$$S_{\text{DBI}}^{D3}(\text{self-dual}) = -T_3 \int d^4x \text{Str} \left[\left(\frac{\hat{r}^2}{R^2} \right)^2 + \frac{1}{4} \theta^2 \right]. \quad (2.46)$$

Moreover, by inspecting eqs. (2.40) and (2.46) we discover that the WZ action cancels against the first term of the right-hand side of (2.46), in complete analogy to what happens to the D7-brane. Thus, one has:

$$S^{D3}(\text{self-dual}) = -\frac{T_3}{4} \int d^4x \text{Str} [\theta^2] = -\pi^2 T_7 (2\pi\alpha')^2 \int d^4x \text{Str} [\theta^2], \quad (2.47)$$

where, in the last step, we have rewritten the result in terms of the tension of the D7-brane. Moreover, an important piece of information is obtained by comparing the WZ terms of the D7- and D3-branes (eqs. (2.28) and (2.40)). Actually, from this comparison we can establish a map between matrices in the D3-brane description and functions of the y coordinates in the D7-brane approach. Indeed, let us suppose that \hat{f} is a $k \times k$ matrix and let us call $f(y)$ the function to which \hat{f} is mapped. It follows from the identification between the D3- and D7-brane WZ actions that the mapping rule is:

$$\text{Str}[\hat{f}] \Rightarrow \int d^4y \mathcal{P}(y) f(y), \quad (2.48)$$

where the kernel $\mathcal{P}(y)$ on the right-hand side of (2.48) is the Pontryagin density defined in eq. (2.26). Actually, the comparison between both WZ actions tells us that the matrix \hat{r}^2 is mapped to the function $\vec{y}^2 + \vec{z}^2$. Notice also that, when \hat{f} is the unit $k \times k$ matrix and $f(y) = 1$, both sides of (2.48) are equal to the instanton number k (see eq. (2.27)). Another interesting information comes by comparing the complete actions of the D3- and D7-branes. It is clear from (2.47) and (2.25) that:

$$(2\pi\alpha')^2 \text{Str}[\theta^2] \Rightarrow \int d^4y \frac{N_f}{\pi^2}. \quad (2.49)$$

By comparing eq. (2.49) with the general relation (2.48), one gets the function that corresponds to the matrix θ^2 , namely:

$$(2\pi\alpha')^2 \theta^2 \Rightarrow \frac{N_f}{\pi^2 \mathcal{P}(y)}. \quad (2.50)$$

Notice that θ^2 is a measure of the non-commutativity of the adjoint scalars in the dielectric approach, i.e. is a quantity that characterizes the fuzziness of the space transverse to the D3-branes. Eq. (2.50) is telling us that this fuzziness is related to the (inverse of the) Pontryagin density for the macroscopic D7-branes. Actually, this identification is reminiscent of the one found in ref. [38] between the non-commutative parameter and the NSNS B-field in the string theory realization of non-commutative geometry. Interestingly, in our case the commutator matrix θ is related to the VEV of the matter fields q and \tilde{q} through the F- and D-flatness conditions (2.9) and (2.10). Notice that eq. (2.50) implies that the quark VEV is somehow related to the instanton density on the flavor brane. In order to make this correspondence more precise, let us consider the one-instanton configuration of the $N_f = 2$ gauge theory on the D7-brane worldvolume. In the so-called singular gauge, the SU(2) gauge field is given by:

$$\frac{A_i}{2\pi\alpha'} = 2i\Lambda^2 \frac{\bar{\sigma}_{ij} y^j}{\rho^2(\rho^2 + \Lambda^2)}, \tag{2.51}$$

where $\rho^2 = \vec{y} \cdot \vec{y}$, Λ is a constant (the instanton size) and the matrices $\bar{\sigma}_{ij}$ are defined as:

$$\bar{\sigma}_{ij} = \frac{1}{4} (\bar{\sigma}_i \sigma_j - \bar{\sigma}_j \sigma_i), \quad \sigma_i = (i\vec{\tau}, 1_{2 \times 2}), \quad \bar{\sigma}_i = \sigma_i^\dagger = (-i\vec{\tau}, 1_{2 \times 2}). \tag{2.52}$$

In (2.52) the $\vec{\tau}$'s are the Pauli matrices. Notice that we are using a convention in which the SU(2) generators are hermitian as a consequence of the relation $\bar{\sigma}_{ij}^\dagger = -\bar{\sigma}_{ij}$. The non-abelian field strength F_{ij} for the gauge potential A_i in (2.51) can be easily computed, with the result:

$$\frac{F_{ij}}{2\pi\alpha'} = -\frac{4i\Lambda^2}{(\rho^2 + \Lambda^2)^2} \bar{\sigma}_{ij} - \frac{8i\Lambda^2}{\rho^2(\rho^2 + \Lambda^2)^2} (y^i \bar{\sigma}_{jk} - y^j \bar{\sigma}_{ik}) y^k. \tag{2.53}$$

Using the fact that the matrices $\bar{\sigma}_{ij}$ are anti self-dual one readily verifies that F_{ij} is self-dual. Moreover, one can prove that:

$$\frac{F_{ij} F_{ij}}{(2\pi\alpha')^2} = \frac{48\Lambda^4}{(\rho^2 + \Lambda^2)^4}, \tag{2.54}$$

which gives rise to the following instanton density:

$$\mathcal{P}(y) = \frac{6}{\pi^2} \frac{\Lambda^4}{(\rho^2 + \Lambda^2)^4}. \tag{2.55}$$

As a check one can verify that eq. (2.27) is satisfied with $k = 1$.

Let us now use this result in (2.50) to get some qualitative understanding of the relation between the Higgs mechanism in field theory and the instanton density in its holographic description. For simplicity we will assume that all quark VEVs are proportional to some scale v , i.e. that:

$$q, \tilde{q} \sim v. \tag{2.56}$$

Then, it follows from (2.45) that:

$$\theta \sim \alpha' v^2, \tag{2.57}$$

and, by plugging this result in (2.50) one arrives at the interesting relation:

$$v \sim \frac{\rho^2 + \Lambda^2}{\alpha' \Lambda} . \tag{2.58}$$

Eq. (2.58) should be understood in the holographic sense, i.e. ρ should be regarded as the energy scale of the gauge theory. Actually, in the far IR ($\rho \approx 0$) the relation (2.58) reduces to:

$$v \sim \frac{\Lambda}{\alpha'} , \tag{2.59}$$

which, up to numerical factors, is precisely the relation between the quark VEV and the instanton size that has been obtained in [26]. Let us now consider the full expression (2.58) for v . For any finite non-zero ρ the quark VEV v is non-zero. Indeed, in both the large and small instanton limits v goes to infinity. However, in the far IR a subtlety arises, since there the quark VEV goes to zero in the small instanton limit. This region should be clearly singular, because a zero quark VEV would mean to uniggs the theory, which would lead to the appearance of extra light degrees of freedom.

To finish this subsection, let us notice that the dielectric effect considered here is not triggered by the influence of any external field other than the metric background. In this sense it is an example of a purely gravitational dielectric effect, as in [39].

2.3 Fluctuations in Dp-D(p+4) with flux

So far we have seen how we can explicitly map the Higgs phase of the field theory to the instanton moduli space in the D7-brane picture through the dielectric description. In this section we will concentrate on the macroscopical description and we will consider fluctuations around the instanton configuration. These fluctuations should correspond to mesons in the dual field theory.

Since we have a similar situation for all the Dp-D(p+4) intersections, namely a one to one correspondence between the Higgs phase of the corresponding field theory and the moduli space of instantons in 4 dimensions, in this section we will work with the general Dp-D(p+4) system. Both the macroscopic and the microscopic analysis of the previous section can be extended in a straightforward manner to the general case, so we will briefly sketch the macroscopical computation to set notations, and concentrate on the fluctuations. In general, the metric corresponding to a stack of N Dp-branes in string frame is given by:

$$ds^2 = \left(\frac{r^2}{R^2} \right)^\alpha dx_{1,p}^2 + \left(\frac{R^2}{r^2} \right)^\alpha d\vec{r}^2 , \quad \alpha = \frac{7-p}{4} , \tag{2.60}$$

where \vec{r} is a $(9-p)$ -dimensional vector and R is given by:

$$R^{7-p} = 2^{5-p} \pi^{\frac{5-p}{2}} \Gamma\left(\frac{7-p}{2}\right) g_s N (\alpha')^{\frac{7-p}{2}} . \tag{2.61}$$

In addition, the type II background generated by the Dp-branes is endowed with a non-zero dilaton given by:

$$e^{-\Phi} = \left(\frac{R^2}{r^2} \right)^\gamma , \quad \gamma = \frac{(7-p)(p-3)}{8} , \tag{2.62}$$

and there is also a RR $(p + 1)$ -form potential, whose expression is:

$$C^{(p+1)} = \left(\frac{r^2}{R^2} \right)^{2\alpha} dx^0 \wedge \cdots \wedge dx^p, \quad (2.63)$$

where α is the same as in eq. (2.60). We will separate again the \vec{r} coordinates in two sets, namely $\vec{r} = (\vec{y}, \vec{z})$, where \vec{y} has four components, and we will denote $\rho^2 = \vec{y} \cdot \vec{y}$. As $r^2 = \rho^2 + \vec{z}^2$, the metric (2.60) can be written as:

$$ds^2 = \left(\frac{\rho^2 + \vec{z}^2}{R^2} \right)^\alpha dx_{1,p}^2 + \left(\frac{R^2}{\rho^2 + \vec{z}^2} \right)^\alpha (d\vec{y}^2 + d\vec{z}^2). \quad (2.64)$$

In this background we will consider a stack of N_f D(p+4)-branes extended along (x^μ, \vec{y}) at fixed distance L in the transverse space spanned by the \vec{z} coordinates (i.e. with $|\vec{z}| = L$). If $\xi^a = (x^\mu, \vec{y})$ are the worldvolume coordinates, the action of a probe D(p+4)-brane is:

$$S^{D(p+4)} = -T_{p+4} \int d^{p+5}\xi e^{-\phi} \text{Str} \left\{ \sqrt{-\det(g + F)} \right\} + \frac{T_{p+4}}{2} \int \text{Str} \left\{ P \left(C^{(p+1)} \right) \wedge F \wedge F \right\}, \quad (2.65)$$

where g is the induced metric and F is the $SU(N_f)$ worldvolume gauge field strength. In order to write g more compactly, let us define the function h as follows:

$$h(\rho) \equiv \left(\frac{R^2}{\rho^2 + L^2} \right)^\alpha. \quad (2.66)$$

Then, one can write the non-vanishing elements of the induced metric as:

$$g_{x^\mu x^\nu} = \frac{\eta_{\mu\nu}}{h}, \quad g_{y^i y^j} = h \delta_{ij}. \quad (2.67)$$

Let us now assume that the only non-vanishing components of the worldvolume gauge field F are those along the y^i coordinates. Following the same steps as in subsection 2.1, the action for the D(p+4)-brane probe can be written as:

$$S^{D(p+4)} = -T_{p+4} \int d^4x d^4y \text{Str} \left\{ \sqrt{1 + \frac{1}{2} \left(\frac{\rho^2 + L^2}{R^2} \right)^{2\alpha} F^2 + \frac{1}{16} \left(\frac{\rho^2 + L^2}{R^2} \right)^{4\alpha} (*FF)^2 - \frac{1}{4} \left(\frac{\rho^2 + L^2}{R^2} \right)^{2\alpha} *FF} \right\}, \quad (2.68)$$

where F^2 and $*FF$ are defined as in eqs. (2.18) and (2.19). If, in addition, F_{ij} is self-dual, one can check that the equations of motion of the gauge field are satisfied and, actually, there is a cancellation between the DBI and WZ parts of the action (2.68) generalizing (2.25), namely:

$$S^{D(p+4)}(\text{self-dual}) = -T_{p+4} \int \text{Str}[1] = -N_f T_{p+4} \int d^{p+1}x \int d^4y. \quad (2.69)$$

We turn now to the analysis of the fluctuations around the self-dual configuration and the computation of the corresponding meson spectrum for this fluxed Dp-D(p+4) intersection. We will not compute the whole set of excitations. Instead, we will focus on the fluctuations of the worldvolume gauge field, for which we will write:

$$A = A^{\text{inst}} + a , \tag{2.70}$$

where A^{inst} is the gauge potential corresponding to a self-dual gauge field strength F^{inst} and a is the fluctuation. The total field strength F reads:

$$F_{ab} = F_{ab}^{\text{inst}} + f_{ab} , \tag{2.71}$$

with f_{ab} being given by:

$$f_{ab} = \partial_a a_b - \partial_b a_a + \frac{1}{2\pi\alpha'} [A_a^{\text{inst}}, a_b] + \frac{1}{2\pi\alpha'} [a_a, A_b^{\text{inst}}] + \frac{1}{2\pi\alpha'} [a_a, a_b] , \tag{2.72}$$

where the indices a, b run now over all the worldvolume directions. Next, let us expand the action (2.65) in powers of the field a up to second order. With this purpose in mind, we rewrite the square root in the DBI action as:

$$\sqrt{-\det(g + F^{\text{inst}} + f)} = \sqrt{-\det(g + F^{\text{inst}})} \sqrt{\det(1 + X)} , \tag{2.73}$$

where X is the matrix:

$$X \equiv (g + F^{\text{inst}})^{-1} f . \tag{2.74}$$

We will expand the right-hand side of (2.73) in powers of X by using the equation²:

$$\sqrt{\det(1 + X)} = 1 + \frac{1}{2} \text{Tr} X - \frac{1}{4} \text{Tr} X^2 + \frac{1}{8} (\text{Tr} X)^2 + o(X^3) . \tag{2.75}$$

To apply this expansion to our problem we need to know previously the value of X , which has been defined in eq. (2.74). Let us denote by \mathcal{G} and \mathcal{J} to the symmetric and antisymmetric part of the inverse of $g + F^{\text{inst}}$, i.e.:

$$(g + F^{\text{inst}})^{-1} = \mathcal{G} + \mathcal{J} . \tag{2.76}$$

One can easily compute the matrix elements of \mathcal{G} , with the result:

$$\mathcal{G}^{\mu\nu} = h \eta^{\mu\nu} , \quad \mathcal{G}^{ij} = \frac{h}{H} \delta_{ij} , \tag{2.77}$$

where h has been defined in (2.66) and the function H is given by:

$$H \equiv h^2 + \frac{1}{4} (F^{\text{inst}})^2 . \tag{2.78}$$

Moreover, the non-vanishing elements of \mathcal{J} are:

$$\mathcal{J}^{ij} = -\frac{F_{ij}^{\text{inst}}}{H} . \tag{2.79}$$

²The trace used in eqs. (2.75) and (2.80) should not be confused with the trace over the $SU(N_f)$ indices.

Using these results one can easily obtain the expression of X and the traces of its powers appearing on the right-hand side of (2.75), which are given by:

$$\begin{aligned} \text{Tr } X &= \frac{1}{H} F_{ij}^{\text{inst}} f_{ij} , \\ \text{Tr } X^2 &= -h^2 f_{\mu\nu} f^{\mu\nu} - \frac{2h^2}{H} f_{i\mu} f^{i\mu} - \frac{h^2}{H^2} f_{ij} f^{ij} + \frac{1}{H^2} F_{ij}^{\text{inst}} F_{kl}^{\text{inst}} f^{jk} f^{li} . \end{aligned} \tag{2.80}$$

By using these results we get, after a straightforward computation, the action up to quadratic order in the fluctuations, namely:

$$\begin{aligned} S^{D(p+4)} &= -T_{p+4} \int \text{Str} \left\{ 1 + \frac{H}{4} f_{\mu\nu} f^{\mu\nu} + \frac{1}{2} f_{i\mu} f^{i\mu} + \frac{1}{4H} f_{ij} f^{ij} + \right. \\ &\quad \left. + \frac{1}{8h^2 H} (F^{ij} f_{ij})^2 - \frac{1}{4h^2 H} F^{ij} F^{kl} f_{jk} f_{li} - \frac{1}{8h^2} f_{ij} f_{kl} \epsilon^{ijkl} \right\} , \end{aligned} \tag{2.81}$$

where we are dropping the superscript in the instanton field strength.

From now on we will assume again that $N_f = 2$ and that the unperturbed configuration is the one-instanton $SU(2)$ gauge field written in eq. (2.51). As in ref. [26], we will concentrate on the subset of fluctuations for which $a_i = 0$, i.e. on those for which the fluctuation field a has non-vanishing components only along the Minkowski directions. However, we should impose this ansatz at the level of the equations of motion in order to ensure the consistency of the truncation. Let us consider first the equation of motion for a_i , which after imposing $a_i = 0$ reduces to:

$$D_i \partial^\mu a_\mu = 0 . \tag{2.82}$$

Moreover, the equation for a_μ when $a_i = 0$ becomes:

$$H D^\mu f_{\mu\nu} + D^i f_{i\nu} = 0 , \tag{2.83}$$

where now H is given in (2.78), with $(F^{\text{inst}})^2$ as in (2.54). Eq. (2.82) is solved by requiring:

$$\partial^\mu a_\mu = 0 . \tag{2.84}$$

Using this result, eq. (2.83) can be written as:

$$H \partial^\mu \partial_\mu a_\nu + \partial_i \partial_i a_\nu + \partial^i \left[\frac{A_i}{2\pi\alpha'}, a_\nu \right] + \left[\frac{A_i}{2\pi\alpha'}, \partial_i a_\nu \right] + \left[\frac{A_i}{2\pi\alpha'}, \left[\frac{A_i}{2\pi\alpha'}, a_\nu \right] \right] = 0 . \tag{2.85}$$

Let us now adopt the following ansatz for a_μ :

$$a_\mu^{(l)} = \xi_\mu(k) f(\rho) e^{ik_\mu x^\mu} \tau^l , \tag{2.86}$$

where τ^l is a Pauli matrix. This ansatz solves eq. (2.84) provided the following transversality condition is fulfilled:

$$k^\mu \xi_\mu = 0 . \tag{2.87}$$

Moreover, one can check that, for this ansatz, one has:

$$\begin{aligned} \partial^i [A_i, a_\nu^{(l)}] &= [A_i, \partial_i a_\nu^{(l)}] = 0, \\ \left[\frac{A_i}{2\pi\alpha'}, \left[\frac{A_i}{2\pi\alpha'}, a_\nu^{(l)} \right] \right] &= -\frac{8\Lambda^4}{\rho^2(\rho^2 + \Lambda^2)^2} \xi_\nu(k) f(\rho) e^{ik_\mu x^\mu} \tau^l. \end{aligned} \quad (2.88)$$

Let us now use these results in eq. (2.85). Denoting $M^2 = -k^2$ (which will be identified with the mass of the meson in the dual field theory) and using eq. (2.54) to compute the function H (see eq. (2.78)), one readily reduces (2.85) to the following second-order differential equation for the function $f(\rho)$ of the ansatz (2.86):

$$\left[\frac{R^{4\alpha} M^2}{(\rho^2 + L^2)^{2\alpha}} \left(1 + \frac{12(2\pi\alpha')^2 \Lambda^4 (\rho^2 + L^2)^{2\alpha}}{R^{4\alpha} (\rho^2 + \Lambda^2)^4} \right) - \frac{8\Lambda^4}{\rho^2(\rho^2 + \Lambda^2)^2} + \frac{1}{\rho^3} \partial_\rho (\rho^3 \partial_\rho) \right] f = 0. \quad (2.89)$$

In order to analyze eq. (2.89), let us introduce a new radial variable ϱ and a reduced mass \bar{M} , which are related to ρ and M as:

$$\rho = L\varrho, \quad \bar{M}^2 = R^{7-p} L^{p-5} M^2. \quad (2.90)$$

Moreover, it is interesting to rewrite the fluctuation equation in terms of field theory quantities. Accordingly, let us introduce the quark mass m_q and its VEV v as follows:

$$m_q = \frac{L}{2\pi\alpha'}, \quad v = \frac{\Lambda}{2\pi\alpha'}. \quad (2.91)$$

Notice that the relation between v and the instanton size Λ is consistent with our analysis of subsection 2.2 (see eq. (2.59)) and with the proposal of ref. [26]. On the other hand, the Yang-Mills coupling g_{YM} is given by:

$$g_{\text{YM}}^2 = (2\pi)^{p-2} (\alpha')^{\frac{p-3}{2}} g_s, \quad (2.92)$$

and the effective dimensionless coupling $g_{\text{eff}}(U)$ at the energy scale U is [40]:

$$g_{\text{eff}}^2(U) = g_{\text{YM}}^2 N U^{p-3}. \quad (2.93)$$

It is now straightforward to use these definitions to rewrite eq. (2.89) as:

$$\left[\frac{\bar{M}^2}{(1 + \varrho^2)^{2\alpha}} \left(1 + c_p(v, m_q) \frac{(1 + \varrho^2)^{2\alpha}}{(\varrho^2 + (\frac{v}{m_q})^2)^4} \right) - \left(\frac{v}{m_q} \right)^4 \frac{8}{\varrho^2(\varrho^2 + (\frac{v}{m_q})^2)^2} + \frac{1}{\varrho^3} \partial_\varrho (\varrho^3 \partial_\varrho) \right] f = 0, \quad (2.94)$$

where $c_p(v, m_q)$ is defined as:

$$c_p(v, m_q) \equiv \frac{12 \cdot 2^{p-2} \pi^{\frac{p+1}{2}}}{\Gamma(\frac{7-p}{2})} \frac{v^4}{g_{\text{eff}}^2(m_q) m_q^4}. \quad (2.95)$$

Notice that everything conspires to absorb the powers of α' and give rise to the effective coupling at the quark mass in $c_p(v, m_q)$.

The equation (2.94) differs in the \bar{M} term from the one obtained in [26], where the term proportional to $c_p(v, m_q)$ is absent. We would like to point out that in order to arrive to (2.94) we expanded up to quadratic order in the fluctuations and we have kept all orders in the instanton field. The extra factor compared to that in ([26]) comes from the fact that, for a self-dual worldvolume gauge field, the unperturbed DBI action actually contains the square root of a perfect square, which can be evaluated exactly and shows up in the lagrangian of the fluctuations. This extra term is proportional to the inverse of the effective Yang-Mills coupling. In order to trust the supergravity approximation the effective Yang-Mills coupling should be large, which would suggest that the effect of this term is indeed negligible. We will see however that in the region of small $\frac{v}{m_q}$ the full term is actually dominating in the IR region and determines the meson spectrum. In addition, in order to ensure the validity of the DBI approximation, we should have slowly varying gauge fields, which further imposes that $F \wedge F$ should be much smaller than α' .

In order to study the fluctuation equation (2.94) it is interesting to notice that, after a change of variable, (2.94) can be converted into a Schrödinger equation. Indeed, let us change from ϱ and f to the new variables z and ψ , defined as:

$$e^z = \varrho, \quad \psi = \varrho f. \tag{2.96}$$

Notice that $\varrho \rightarrow \infty$ corresponds to $z \rightarrow +\infty$, while $\varrho = 0$ is mapped to $z = -\infty$. Moreover, one can readily prove that, in terms of z and ψ , eq. (2.94) can be recast as:

$$\partial_z^2 \psi - V(z) \psi = 0, \tag{2.97}$$

where the potential $V(z)$ is given by:

$$V(z) = 1 + \left(\frac{v}{m_q}\right)^4 \frac{8}{\left(e^{2z} + \left(\frac{v}{m_q}\right)^2\right)^2} - \bar{M}^2 \frac{e^{2z}}{\left(e^{2z} + 1\right)^{\frac{7-p}{2}}} \left[1 + c_p(v, m_q) \frac{\left(e^{2z} + 1\right)^{\frac{7-p}{2}}}{\left(e^{2z} + \left(\frac{v}{m_q}\right)^2\right)^4} \right]. \tag{2.98}$$

Notice that the reduced mass \bar{M} is just a parameter in $V(z)$. Actually, in these new variables the problem of finding the mass spectrum can be rephrased as that of finding the values of \bar{M} that allow a zero-energy level for the potential (2.98). By using the standard techniques in quantum mechanics one can convince oneself that such solutions exist. Indeed, the potential (2.98) is strictly positive for $z \rightarrow \pm\infty$ and has some minima for finite values of z . The actual calculation of the mass spectra must be done by means of numerical techniques. A key ingredient in this approach is the knowledge of the asymptotic behaviour of the solution when $\varrho \rightarrow 0$ and $\varrho \rightarrow \infty$. This behaviour can be easily obtained from the form of the potential $V(z)$ in (2.98). Indeed, for $\varrho \rightarrow \infty$, or equivalently for $z \rightarrow +\infty$, the potential $V(z) \rightarrow 1$, and the solutions of (2.97) behave as $\psi \sim e^{\pm z}$ which, in terms of the original variables, corresponds to $f = \text{constant}$, ϱ^{-2} . Similarly for $\varrho \rightarrow 0$ (or

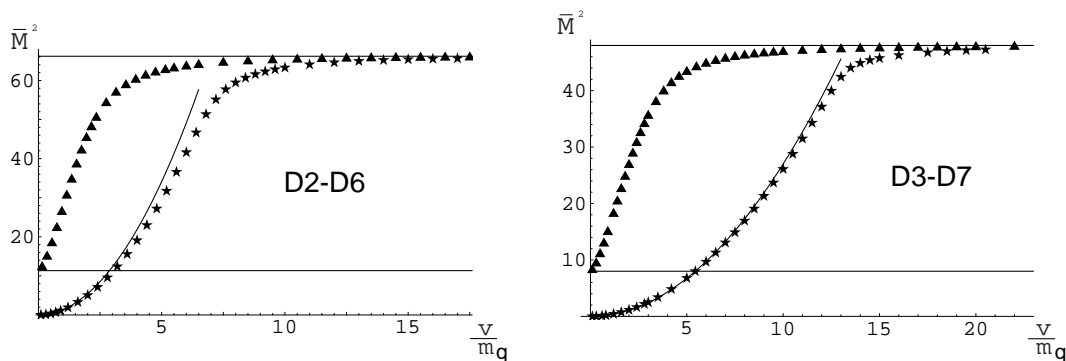


Figure 1: In this figure we plot the numerical masses for the first level as a function of the instanton size for both the full equation (with stars) and for the equation obtained in [26] (with solid triangles). The quark mass m_q is such that $g_{\text{eff}}(m_q) = 1$. The solid line corresponds to the WKB prediction (2.107) for small v . The plot on the left (right) corresponds to the D2-D6 (D3-D7) intersection.

$z \rightarrow -\infty$) one gets that $f = \varrho^2, \varrho^{-4}$. Thus, the IR and UV behaviours of the fluctuation are:

$$\begin{aligned}
 f(\varrho) &\approx a_1 \varrho^2 + a_2 \varrho^{-4}, & (\varrho \rightarrow 0), \\
 f(\varrho) &\approx b_1 \varrho^{-2} + b_2, & (\varrho \rightarrow \infty).
 \end{aligned}
 \tag{2.99}$$

The normalizable solutions are those that are regular at $\varrho \approx 0$ and decrease at $\varrho \approx \infty$. Thus they correspond to having $a_2 = b_2 = 0$ in (2.99). Upon applying a shooting technique, we can determine the values of \bar{M} for which such normalizable solutions exist. Notice that \bar{M} depends parametrically on the quark mass m_q and on its VEV v . In general, for given values of m_q and v , one gets a tower of discrete values of \bar{M} . In figure 1 we have plotted the values of the reduced mass for the first level, as a function of the quark VEV. For illustrative purposes we have included the values obtained with the fluctuation equation of ref. [26]. As anticipated above, both results differ significantly in the region of small v and coincide when $v \rightarrow \infty$. Actually, when v is very large we recover the spectral flow phenomenon described in [26], i.e. \bar{M} becomes independent of the instanton size and equals the mass corresponding to a higher Kaluza-Klein mode on the worldvolume sphere. However, we see that when $\frac{v}{m_q}$ goes to zero, the masses of the associated fluctuations also go to zero. Actually, this limit is pretty singular. Indeed, it corresponds to the small instanton limit, where it is expected that the moduli space of instantons becomes effectively non-compact and that extra massless degrees of freedom show up in the spectrum.

It turns out that the mass levels for small v are nicely represented analytically by means of the WKB approximation for the Schrödinger problem (2.97). The WKB method has been very successful [41, 42] in the calculation of the glueball mass spectra in the gauge/gravity correspondence and also provides rather reliable predictions for the mass

levels of the mesons [20]. The WKB quantization rule is:

$$\left(n + \frac{1}{2}\right)\pi = \int_{z_1}^{z_2} dz \sqrt{-V(z)}, \quad n \geq 0, \quad (2.100)$$

where $n \in \mathbb{Z}$ and z_1 and z_2 are the turning points of the potential ($V(z_1) = V(z_2) = 0$). Following straightforwardly the steps of refs. [42, 20], we obtain the following expression for the WKB values of \bar{M} :

$$\bar{M}_{\text{WKB}}^2 = \frac{\pi^2}{\zeta^2} (n+1) \left(n + 3 + \frac{2}{5-p}\right), \quad (2.101)$$

where ζ is the following integral:

$$\zeta = \int_0^{+\infty} d\varrho \sqrt{\frac{1}{(1+\varrho^2)^{\frac{7-p}{2}}} + \frac{c_p(v, m_q)}{\left[\left(\frac{v}{m_q}\right)^2 + \varrho^2\right]^4}}. \quad (2.102)$$

Let us evaluate analytically ζ when v is small. First of all, as can be easily checked, we notice that, when v is small, the second term under the square root in (2.102) behaves as:

$$\frac{1}{\left[\left(\frac{v}{m_q}\right)^2 + \varrho^2\right]^2} \approx \frac{\pi}{2} \left(\frac{m_q}{v}\right)^3 \delta(\varrho), \quad \text{as } v \rightarrow 0. \quad (2.103)$$

Then, one can see that this term dominates the integral defining ζ around $\varrho \approx 0$ and, for small v , one can approximate ζ as:

$$\zeta \approx \frac{\sqrt{c_p(v, m_q)}}{2} \int_{-\epsilon}^{\epsilon} \frac{d\varrho}{\left[\left(\frac{v}{m_q}\right)^2 + \varrho^2\right]^2} + \int_0^{+\infty} \frac{d\varrho}{(1+\varrho^2)^{\frac{7-p}{4}}}, \quad (2.104)$$

where ϵ is a small positive number and we have used the fact that the function in (2.102) is an even function of ϱ . Using (2.103), one can evaluate ζ as:

$$\zeta \approx \frac{\pi}{4} \left(\frac{m_q}{v}\right)^3 \sqrt{c_p(v, m_q)} + \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(\frac{5-p}{4}\right)}{\Gamma\left(\frac{7-p}{4}\right)}. \quad (2.105)$$

Clearly, for $v \rightarrow 0$, we can neglect the last term in (2.105). Using the expression of $c_p(v, m_q)$ (eq. (2.95)), we arrive at:

$$\zeta \approx \frac{\sqrt{3} \cdot 2^{\frac{p-4}{2}} \pi^{\frac{p+5}{4}}}{\sqrt{\Gamma\left(\frac{7-p}{2}\right)}} \frac{m_q}{g_{\text{eff}}(m_q) v}, \quad (2.106)$$

and plugging this result in (2.101), we get the WKB mass of the ground state ($n = 0$) for small v :

$$\bar{M}_{\text{WKB}}^2 \approx \frac{(17-3p)\Gamma\left(\frac{5-p}{2}\right)}{3 \cdot 2^{p-3} \pi^{\frac{p+1}{2}}} \left(\frac{g_{\text{eff}}(m_q) v}{m_q}\right)^2. \quad (2.107)$$

Thus, we predict that \bar{M}^2 is a quadratic function of v/m_q with the particular coefficient given on the right-hand side of (2.107). In figure 1 we have represented by a solid line the value of \bar{M} obtained from eq. (2.107). We notice that, for small v , this equation nicely fits the values obtained by the numerical calculation.

Let us now study the dependence of the mass gap as a function of the quark mass m_q and the quark VEV v . First of all, we notice that the relation between the reduced mass \bar{M} and the mass M can be rewritten in terms of the quark mass m_q and the dimensionless coupling constant $g_{\text{eff}}(m_q)$ as:

$$M \propto \frac{m_q}{g_{\text{eff}}(m_q)} \bar{M} . \tag{2.108}$$

For large v the reduced mass \bar{M} tends to a value independent of both m_q and v . Thus, the meson mass M depends only on m_q in a holographic way, namely:

$$M \sim \frac{m_q}{g_{\text{eff}}(m_q)} , \quad (v \rightarrow \infty) . \tag{2.109}$$

Notice that this dependence on m_q and v is exactly the same as in the unbroken symmetry case, although the numerical coefficient is different from that found in [20, 21]. On the contrary, for small v , after combining eq. (2.108) with the WKB result (2.107), we get that the mass gap depends linearly on v and is independent on the quark mass m_q :

$$M \sim v , \quad (v \rightarrow 0) , \tag{2.110}$$

and, in particular, the mass gap disappears in the limit $v \rightarrow 0$, which corresponds to having a zero size instanton.

3. The codimension one defect

In this section we will consider the intersection of Dp- and D(p+2)-branes according to the array:

$$\begin{array}{cccccccccccc} & 1 & \cdots & p-1 & p & p+1 & p+2 & p+3 & \cdots & 9 & & \\ Dp : & \times & \cdots & \times & \times & - & - & - & \cdots & - & & \\ D(p+2) : & \times & \cdots & \times & - & \times & \times & \times & \cdots & - & & \end{array} \tag{3.1}$$

It is easy to verify, by using the standard intersection rules of the type II theories, that this Dp-D(p+2) intersection is supersymmetric. Moreover, it is clear from (3.1) that the D(p+2)-brane is an object of codimension one along the gauge theory directions of the Dp-brane worldvolume. Indeed, for $p = 3$ the configuration (3.1) was studied in [3] and shown to be dual to a defect theory in which $\mathcal{N} = 4$, $d = 4$ super Yang-Mills theory in the bulk is coupled to $\mathcal{N} = 4$, $d = 3$ fundamental hypermultiplets localized at the defect [32, 43], which is located at a fixed value of the coordinate p in (3.1). These hypermultiplets are generated by open strings connecting the two types of D-branes. If we allow a non-zero distance in the $p+4, \dots, 9$ directions of the two stacks in (3.1), the hypermultiplets become massive and a mass gap is introduced in the theory. The corresponding meson spectrum was computed in the probe approximation in ref. [20].

The analysis of the Higgs phase of the codimension one defect associated to the array (3.1) has been worked out recently in [31] for the particular case of a 2+1 dimensional defect living in a bulk $\mathcal{N} = 4$, $d = 4$ theory, which corresponds to the intersection displayed in (3.1) for $p = 3$. In that reference it is argued that the gravity dual description of the Higgs phase of the theory with $\mathcal{N} = 2$ fundamental hypermultiplets confined to a codimension one defect is in terms of probe D5-branes in the near horizon of the D3-brane geometry, once we switch on appropriately a magnetic worldvolume gauge field. As in the case of the codimension zero defect, this worldvolume gauge field has the effect of introducing extra D3-brane charge in the D5-brane worldvolume, which in turn can be seen as the macroscopical description of dielectrically expanded D3-branes. However, the addition of the magnetic field requires a non-trivial bending of the D5-branes, which now recombine with the D3's rather than intersecting them. This bending takes place along the direction 3 in (3.1) for $p = 3$. As in the previous section, the F - and D -flatness conditions arise naturally as the vacuum conditions for the dielectric branes, thus providing a map between the Higgs phase of these theories and the monopoles in the sphere to which the branes expand. The required bending then appears naturally as the solution to these F - and D -flatness conditions.

We will refer to [31] for the field theory analysis, which can be extended in a straightforward manner to any dimension. Instead, in this paper we will focus on the gravity dual for the general case, which will be in terms of a probe D(p+2)-brane in the Dp-brane background given by (2.60), (2.62) and (2.63). Let us go to a new coordinate system, in which we write the transverse space to the Dp-brane spanned by \vec{r} in a more suitable manner, such that the metric (2.60) takes the form:

$$ds^2 = \left(\frac{r^2}{R^2}\right)^\alpha dx_{1,p}^2 + \left(\frac{R^2}{r^2}\right)^\alpha \left(d\rho^2 + \rho^2 d\Omega_2^2 + d\vec{z}^2\right), \quad (3.2)$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the line element of a unit two-sphere and the coordinates (ρ, θ, φ) parametrize the directions $p + 1$, $p + 2$ and $p + 3$ in (3.1). The exponent α in (3.2) is the same as in (2.60) and $r^2 = \rho^2 + \vec{z}^2$.

We shall now consider a D(p+2)-brane probe in this background. Its action is:

$$S^{D(p+2)} = -T_{p+2} \int d^{p+3}\xi e^{-\phi} \sqrt{-\det(g + F)} + T_{p+2} \int P[C^{(p+1)}] \wedge F. \quad (3.3)$$

In what follows we will take $\xi^a = (x^0, x^1, \dots, x^{p-1}, \rho, \theta, \varphi)$ as worldvolume coordinates. Moreover, we will assume that there exists a constant separation on the transverse space, $\vec{z}^2 = L^2$, which gives mass to the quarks, and we will switch on a magnetic worldvolume field on the internal S^2 given by:

$$F = q \text{Vol}(S^2) \equiv \mathcal{F}, \quad (3.4)$$

where q is a constant and $\text{Vol}(S^2) = \sin\theta d\theta d\varphi$. As anticipated above, in order to solve the equations of motion of the probe, we have to consider a non-trivial transverse x^p field

$x^p = x(\rho)$. Moreover, since nothing depends on the internal S^2 , upon integration over this compact manifold, it is straightforward to see that the action reads:

$$S^{D(p+2)} = -4\pi T_{p+2} \int d^p x d\rho \left\{ \rho^2 \sqrt{1 + \left(\frac{\rho^2 + L^2}{R^2}\right)^{2\alpha}} x'^2 \sqrt{1 + \left(\frac{\rho^2 + L^2}{R^2}\right)^{2\alpha} \frac{q^2}{\rho^4}} - q \left(\frac{\rho^2 + L^2}{R^2}\right)^{2\alpha} x' \right\}. \quad (3.5)$$

One can check that the Euler-Lagrange equation for $x(\rho)$ derived from (3.5) is solved if one requires that:

$$x'(\rho) = \frac{q}{\rho^2}, \quad (3.6)$$

which can be immediately integrated, giving rise to the following profile of the transverse scalar:

$$x(\rho) = x_0 - \frac{q}{\rho} \equiv \mathcal{X}(\rho). \quad (3.7)$$

For this configuration, the two square roots in (3.5) become equal and there is a cancellation between the WZ and (part of) the DBI term. Then, the energy for such a brane, which is nothing but minus the lagrangian since our configuration is static, reduces to:

$$E = 4\pi T_{p+2} \int \rho^2, \quad (3.8)$$

where, as in the Dp-D(p+4) case, the distance L does not explicitly appear, displaying the supersymmetry properties of the configuration. Indeed, one can verify as in [33] that the condition (3.6) is a BPS equation that can be derived from kappa symmetry of the probe and that the energy (3.8) saturates a BPS bound.

Let us remind the reader that the existence of the bending (3.7) was a key ingredient in the analysis of [31], where it was shown, for the particular case of $p = 3$, that it has the effect of spreading the defect over the whole bulk which, in turn, led to the loss of the discrete spectrum. We shall see that, indeed, the same situation occurs in the more general case considered here.

3.1 Microscopical description of the Dp-D(p+2) intersection with flux

The flux of the worldvolume gauge field F of eq. (3.4) on the internal S^2 has the non-trivial effect of inducing Dp-brane charge in the D(p+2)-brane worldvolume. To verify this fact, let us point out that F is constrained by a flux quantization condition [44] which, with our notations, reads:

$$\int_{S^2} F = \frac{2\pi k}{T_f}, \quad k \in \mathbb{Z}, \quad T_f = \frac{1}{2\pi\alpha'}. \quad (3.9)$$

By plugging the expression of F given in (3.4) on the quantization condition (3.9), one immediately concludes that the constant q is restricted to be of the form:

$$q = k\pi\alpha', \quad (3.10)$$

where k is an integer. In order to interpret the meaning of k , let us notice that in the WZ piece of the action for the D(p+2)-brane we have the coupling:

$$T_{p+2} \int P[C^{p+1}] \wedge F . \tag{3.11}$$

Upon using the explicit form of F to integrate it over the two-sphere and the relation $T_{p+2} (2\pi)^2 \alpha' = T_p$ we have that this coupling reads:

$$k T_p \int P[C^{p+1}] , \tag{3.12}$$

where now the integration is over $p + 1$ variables. Thus, we see that F is inducing k units of Dp-brane charge in the worldvolume of the D(p+2)-brane. This charge is located along the $\{x^0, \dots, x^{p-1}, \rho\}$ directions. This suggests an alternative interpretation of the system in terms of dielectric Dp-branes that polarize to a D(p+2)-brane, as anticipated in [31]. To be more explicit, let us consider a stack of k coincident Dp-branes in the background (3.2). The dynamics of such a stack is governed by the Myers action [25], which is given by the straightforward generalization of eqs. (2.31) and (2.32) to a Dp-brane. We will choose $(x^0, \dots, x^{p-1}, \rho)$ as worldvolume coordinates and we shall consider the other coordinates in the metric (3.2) as scalar fields which, in general, are non-commutative. Moreover, we shall introduce new coordinates $Y^I (I = 1, 2, 3)$ for the two-sphere of the metric (3.2). These new coordinates satisfy $\sum_I Y^I Y^I = 1$ and the line element $d\Omega_2^2$ is given by:

$$d\Omega_2^2 = \sum_I dY^I dY^I , \quad \sum_I Y^I Y^I = 1 . \tag{3.13}$$

We will assume that the Y 's are the only non-commutative scalars and that they are represented by $k \times k$ matrices. Furthermore, we shall adopt the ansatz in which they are given by:

$$Y^I = \frac{J^I}{\sqrt{C_2(k)}} , \tag{3.14}$$

where the $k \times k$ matrices J^I correspond to the k -dimensional irreducible representation of the SU(2) algebra:

$$[J^I, J^J] = 2i\epsilon_{IJK} J^K , \tag{3.15}$$

and $C_2(k) = k^2 - 1$ is the corresponding quadratic Casimir. Clearly the Y 's parametrize a fuzzy two-sphere. Let us, in addition, assume that we consider embeddings of the Dp-brane in which the scalars \vec{z} and x^p are commutative and such that $|\vec{z}| = L$ and $x^p = x(\rho)$. With these assumptions it is easy to evaluate the dielectric action for the Dp-brane in the large k limit, following the same steps as those followed in ref. [31] for the D3-D5 system. The final result exactly coincides with the macroscopical action (3.5), once q is related to the integer k as in the quantization condition (3.10). This matching is a confirmation of our interpretation of the D(p+2)-brane configuration with flux as a bound state of a stack of coincident Dp-branes. Once again we see that the expansion to the dielectric configuration is not caused by any other field apart from the metric background, thus constituting another example of purely gravitational dielectric effect ([39]).

3.2 Fluctuations in Dp-D(p+2) with flux

Let us now study the fluctuations around the Dp-D(p+2) intersection with flux described above. Without loss of generality we can take the unperturbed configuration as $z^1 = L$, $z^m = 0$ ($m > 1$). Next, let us consider a fluctuation of the type:

$$\begin{aligned} z^1 &= L + \chi^1, & z^m &= \chi^m, & (m = 2, \dots, 6-p), \\ x^p &= \mathcal{X} + x, & F &= \mathcal{F} + f, \end{aligned} \tag{3.16}$$

where the bending \mathcal{X} and the worldvolume gauge field \mathcal{F} are given by eqs. (3.7) and (3.4) respectively and we assume that χ^m , x and f are small. The induced metric on the D(p+2)-brane worldvolume can be written as:

$$g = \mathcal{G} + g^{(f)}, \tag{3.17}$$

with \mathcal{G} being the induced metric of the unperturbed configuration:

$$\mathcal{G}_{ab} d\xi^a d\xi^b = h^{-1} dx_{1,p-1}^2 + h \left[\left(1 + \frac{q^2}{\rho^4 h^2} \right) d\rho^2 + \rho^2 d\Omega_2^2 \right], \tag{3.18}$$

where $h = h(\rho)$ is the function defined in (2.66). Moreover, $g^{(f)}$ is the part of g that depends on the derivatives of the fluctuations, namely:

$$g_{ab}^{(f)} = \frac{q}{\rho^2 h} \left(\delta_{a\rho} \partial_b x + \delta_{b\rho} \partial_a x \right) + \frac{1}{h} \partial_a x \partial_b x + h \partial_a \chi^m \partial_b \chi^m. \tag{3.19}$$

Let us next rewrite the Born-Infeld determinant as:

$$\sqrt{-\det(g+F)} = \sqrt{-\det(\mathcal{G} + \mathcal{F})} \sqrt{\det(1+X)}, \tag{3.20}$$

where the matrix X is given by:

$$X \equiv \left(\mathcal{G} + \mathcal{F} \right)^{-1} \left(g^{(f)} + f \right). \tag{3.21}$$

We shall evaluate the right-hand side of (3.20) by expanding it in powers of X by means of eq. (2.75). In order to evaluate more easily the trace of the powers of X appearing on the right-hand side of this equation, let us separate the symmetric and antisymmetric part in the inverse of the matrix $\mathcal{G} + \mathcal{F}$:

$$\left(\mathcal{G} + \mathcal{F} \right)^{-1} = \hat{\mathcal{G}}^{-1} + \mathcal{J}, \tag{3.22}$$

where:

$$\hat{\mathcal{G}}^{-1} \equiv \frac{1}{(\mathcal{G} + \mathcal{F})_S}, \quad \mathcal{J} \equiv \frac{1}{(\mathcal{G} + \mathcal{F})_A}. \tag{3.23}$$

Notice that $\hat{\mathcal{G}}$ is just the open string metric which, for the case at hand, is given by:

$$\hat{\mathcal{G}}_{ab} d\xi^a d\xi^b = h^{-1} dx_{1,p-1}^2 + h \left(1 + \frac{q^2}{\rho^4 h^2} \right) \left(d\rho^2 + \rho^2 d\Omega_2^2 \right). \tag{3.24}$$

Moreover, the antisymmetric matrix \mathcal{J} takes the form:

$$\mathcal{J}^{\theta\varphi} = -\mathcal{J}^{\varphi\theta} = -\frac{1}{\sqrt{\tilde{g}}} \frac{q}{q^2 + \rho^4 h^2}, \quad (3.25)$$

where θ, φ are the standard polar coordinates on S^2 and $\tilde{g} = \sin^2 \theta$ is the determinant of its round metric. It is now straightforward to show that:

$$\text{tr } X = h \hat{\mathcal{G}}^{ab} \partial_a \chi^m \partial_b \chi^m + \frac{1}{h} \hat{\mathcal{G}}^{ab} \partial_a x \partial_b x + \frac{q}{q^2 + \rho^4 h^2} \left[2\rho^2 \partial_\rho x + \frac{\epsilon^{ij} f_{ij}}{\sqrt{\tilde{g}}} \right], \quad (3.26)$$

while, up to quadratic terms in the fluctuations, $\text{tr } X^2$ is given by:

$$\begin{aligned} \text{tr } X^2 = & -f_{ab} f^{ab} + \frac{2}{h} \frac{q^2}{q^2 + \rho^4 h^2} \left[\hat{\mathcal{G}}^{ab} \partial_a x \partial_b x + \hat{\mathcal{G}}^{\rho\rho} (\partial_\rho x)^2 \right] \\ & + \frac{q^2}{(q^2 + \rho^4 h^2)^2} \left[\frac{1}{2\tilde{g}} (\epsilon^{ij} f_{ij})^2 - 4\rho^2 \frac{\epsilon^{ij} \partial_i x f_{j\rho}}{\sqrt{\tilde{g}}} \right], \end{aligned} \quad (3.27)$$

where the indices i, j refer to the directions along the S^2 and $\epsilon^{ij} = \pm 1$. Using these results one can readily compute the DBI term of the lagrangian density. Dropping constant global factors that do not affect the equations of motion, one gets:

$$\begin{aligned} \mathcal{L}_{\text{DBI}} = & -\rho^2 \sqrt{\tilde{g}} \left[1 + \frac{q^2}{\rho^4 h^2} + \frac{h}{2} \left(1 + \frac{q^2}{\rho^4 h^2} \right) \hat{\mathcal{G}}^{ab} \partial_a \chi^m \partial_b \chi^m \right. \\ & \left. + \frac{1}{2h} \hat{\mathcal{G}}^{ab} \partial_a x \partial_b x + \frac{1}{4} \left(1 + \frac{q^2}{\rho^4 h^2} \right) f_{ab} f^{ab} \right] \\ & + \frac{A(\rho)}{2} x \epsilon^{ij} f_{ij} - \frac{q\sqrt{\tilde{g}}}{h^2} \partial_\rho x - \frac{q}{2\rho^2 h^2} \epsilon^{ij} f_{ij}, \end{aligned} \quad (3.28)$$

where the indices a, b are raised with the open string metric $\hat{\mathcal{G}}$ and $A(\rho)$ is the following function:

$$A(\rho) \equiv \frac{d}{d\rho} \left[\frac{q^2}{h^2 (q^2 + \rho^4 h^2)} \right]. \quad (3.29)$$

To get the above expression of \mathcal{L}_{DBI} we have integrated by parts and made use of the Bianchi identity for the gauge field fluctuation:

$$\epsilon^{ij} \partial_i f_{j\rho} + \frac{\epsilon^{ij}}{2} \partial_\rho f_{ij} = 0. \quad (3.30)$$

Similarly, the WZ term can be written as:

$$\mathcal{L}_{\text{WZ}} = \sqrt{\tilde{g}} \frac{q^2}{\rho^2 h^2} + \sqrt{\tilde{g}} \frac{q}{h^2} \partial_\rho x + \frac{q}{2\rho^2 h^2} \epsilon^{ij} f_{ij} + \frac{\partial_\rho h}{h^3} x \epsilon^{ij} f_{ij}. \quad (3.31)$$

By combining \mathcal{L}_{DBI} and \mathcal{L}_{WZ} and dropping the term independent of the fluctuations, we get that the total lagrangian density is given by:

$$\mathcal{L} = -\rho^2 \sqrt{\tilde{g}} \left[\frac{h}{2} \left(1 + \frac{q^2}{\rho^4 h^2} \right) \hat{\mathcal{G}}^{ab} \partial_a \chi^m \partial_b \chi^m + \frac{1}{2h} \hat{\mathcal{G}}^{ab} \partial_a x \partial_b x \right. \\ \left. + \frac{1}{4} \left(1 + \frac{q^2}{\rho^4 h^2} \right) f_{ab} f^{ab} \right] - \frac{C(\rho)}{2} x \epsilon^{ij} f_{ij}. \quad (3.32)$$

In eq. (3.32), and in what follows, the function $C(\rho)$ is given by:

$$C(\rho) \equiv \frac{d}{d\rho} \left[\frac{\rho^4}{q^2 + \rho^4 h^2} \right]. \quad (3.33)$$

As it is manifest from (3.32), the transverse scalars χ do not couple to other fields, while the scalar x is coupled to the fluctuations f_{ij} of the gauge field strength along the two-sphere. For the fluxless case $q = 0$ these equations were solved in ref. [20], where it was shown that they give rise to a discrete meson mass spectrum, which can be computed numerically and, in the case of the D3-D5 intersection, analytically. Let us examine here the situation for $q \neq 0$. The equation of motion of the transverse scalars χ that follows from (3.32) is:

$$\partial_a \left[\sqrt{\tilde{g}} \rho^2 h \left(1 + \frac{q^2}{\rho^4 h^2} \right) \hat{\mathcal{G}}^{ab} \partial_b \chi \right] = 0. \quad (3.34)$$

By using the explicit form of the open string metric $\hat{\mathcal{G}}^{ab}$ (eq. (3.24)), we can rewrite (3.34) as:

$$\partial_\rho \left(\rho^2 \partial_\rho \chi \right) + \left[\rho^2 h^2 + \frac{q^2}{\rho^2} \right] \partial^\mu \partial_\mu \chi + \nabla^i \nabla_i \chi = 0. \quad (3.35)$$

Let us separate variables and write the scalars in terms of the eigenfunctions of the laplacian in the Minkowski and sphere parts of the brane geometry as:

$$\chi = e^{ikx} Y^l(S^2) \xi(\rho), \quad (3.36)$$

where the product kx is performed with the Minkowski metric and l is the angular momentum on the S^2 . The fluctuation equation for the function ξ is:

$$\partial_\rho \left(\rho^2 \partial_\rho \xi \right) + \left\{ \left[R^{4\alpha} \frac{\rho^2}{(\rho^2 + L^2)^{2\alpha}} + \frac{q^2}{\rho^2} \right] M^2 - l(l+1) \right\} \xi = 0, \quad (3.37)$$

where $M^2 = -k^2$ is the mass of the meson. When the distance $L \neq 0$ and $q = 0$ eq. (3.37) gives rise to a set of normalizable solutions that occur for a discrete set of values of M [20]. As argued in ref. [31] for the D3-D5 system, the situation changes drastically when the flux is switched on. Indeed, let us consider the equation (3.37) when $L, q \neq 0$ in the IR, i.e. when ρ is close to zero. In this case, for small values of ρ , eq. (3.37) reduces to:

$$\partial_\rho \left(\rho^2 \partial_\rho \xi \right) + \left[\frac{q^2 M^2}{\rho^2} - l(l+1) \right] \xi = 0, \quad (\rho \approx 0). \quad (3.38)$$

Eq. (3.38) can be solved in terms of Bessel functions, namely:

$$\xi = \frac{1}{\sqrt{\rho}} J_{\pm(l+\frac{1}{2})} \left(\frac{qM}{\rho} \right), \quad (\rho \approx 0). \quad (3.39)$$

Near $\rho \approx 0$ the Bessel function (3.39) oscillates infinitely as:

$$\xi \approx e^{\pm i \frac{qM}{\rho}}, \quad (\rho \approx 0). \quad (3.40)$$

The behaviour (3.40) implies that the spectrum of M is continuous and gapless. Actually, one can understand this result by rewriting the function (3.39) in terms of the coordinate x^p by using (3.7). Indeed, $\rho \approx 0$ corresponds to large $|x^p|$ and $\xi(x^p)$ can be written in this limit as a simple plane wave:

$$\xi \approx e^{\pm i M x^p}, \quad (|x^p| \rightarrow \infty). \quad (3.41)$$

Thus, the fluctuation spreads out of the defect locus at fixed x^p , reflecting the fact that the bending has the effect of recombining, rather than intersecting, the Dp-branes with the D(p+2)-branes. As in ref. [31] we can understand this result by looking at the IR form of the open string metric (3.24). One gets:

$$\hat{\mathcal{G}}_{ab} d\xi^a d\xi^b \approx \frac{L^{2\alpha}}{R^{2\alpha}} \left[dx_{1,p-1}^2 + q^2 \left(\frac{d\rho^2}{\rho^4} + \frac{1}{\rho^2} d\Omega_2^2 \right) \right], \quad (\rho \approx 0). \quad (3.42)$$

By changing variables from ρ to $u = q/\rho$, this metric can be written as:

$$\frac{L^{2\alpha}}{R^{2\alpha}} \left[dx_{1,p-1}^2 + du^2 + u^2 d\Omega_2^2 \right], \quad (3.43)$$

which is nothing but the (p+3)-dimensional Minkowski space and, thus, one naturally expects to get plane waves as in (3.41) as solutions of the fluctuation equations. This fact is generic for all the fluctuations of this system. Recall that the other fields in the Lagrangian (3.32) are coupled. However, in appendix A we show that they can be decoupled by generalizing the results of ref. [32, 20]. The decoupled fluctuation equations can actually be mapped [21] to that satisfied by the scalars χ . Thus, we conclude that the full mesonic mass spectrum is continuous and gapless, as a consequence of the recombination of the color and flavor branes induced by the worldvolume flux.

3.3 An S-dual picture: the F1-Dp intersection

Let us now have a look to the S-dual configurations for the IIB cases in this section, which will give us information about the weak 't Hooft coupling regime of the dual theory. For $p = 3$ the S-dual background will be once again $AdS_5 \times S^5$. In this case, the D5-brane gets mapped to a NS5 brane. However, since the dilaton is zero in this background, at least formally this situation will be identical to the D3-D5 case already studied above. In particular we will lose again the discrete spectrum. In turn, we can look at the $p = 1$ case,

whose S-dual version is the F1-D3 intersection. Actually, we will analyze the more general system corresponding to the F1-Dp intersection, according to the array:

$$\begin{array}{cccccccc}
 & 1 & 2 & \cdots & p+1 & p+2 & \cdots & 9 \\
 F1 : & \times & - & \cdots & - & - & \cdots & - \\
 Dp : & - & \times & \cdots & \times & - & \cdots & -
 \end{array} \tag{3.44}$$

As in previous cases, we will consider a stack of F1 strings, which we will treat as a background of type II theory. The corresponding metric is given by:

$$ds^2 = H^{-1} dx_{1,1}^2 + d\vec{r}^2, \tag{3.45}$$

where, in the near-horizon limit, $H = R^6/r^6$, with $R^6 = 32\pi^2(\alpha')^3 g_s^2 N$. The F1 background is also endowed with a NSNS B field and a non-trivial dilaton, given by:

$$B = H^{-1} dx_0 \wedge dx_1, \quad e^{-\Phi} = H^{\frac{1}{2}}. \tag{3.46}$$

Let us now rewrite this solution in terms of a new coordinate system more suitable for our probe analysis. First of all, we split the coordinates transverse to the F1 as $\vec{r} = (\vec{y}, \vec{z})$, where the \vec{y} vector corresponds to the directions $2 \cdots p+1$ and \vec{z} refers to the coordinates transverse to both the F1 and Dp-brane. Moreover, let us assume that $p > 1$ and use spherical coordinates to parametrize the subspace spanned by the y 's, i.e. $d\vec{y}^2 = d\rho^2 + \rho^2 d\Omega_{p-1}^2$. Then, the metric (3.45) can be rewritten as:

$$ds^2 = H^{-1} dx_{1,1}^2 + d\rho^2 + \rho^2 d\Omega_{p-1}^2 + d\vec{z}^2. \tag{3.47}$$

The dynamics of the Dp-brane probe is determined by the DBI lagrangian, which in this case takes the form:

$$\mathcal{L} = -T_p e^{-\phi} \sqrt{-\det(g + \mathcal{F})}, \tag{3.48}$$

where \mathcal{F} is the following combination of the worldvolume gauge field strength F and the pullback $P[B]$ of the NSNS B field:

$$\mathcal{F} = F - P[B]. \tag{3.49}$$

Let us choose x^0, ρ and the $p-1$ angles parametrizing the S^{p-1} sphere as our set of worldvolume coordinates. We will consider embeddings of the type:

$$x^1 = x(\rho), \quad |\vec{z}| = L. \tag{3.50}$$

Moreover, we will switch on an electric field $F_{0\rho} \equiv F$ in the worldvolume, such that the only non-vanishing component of \mathcal{F} is:

$$\mathcal{F}_{0\rho} = F - H^{-1} x', \tag{3.51}$$

where, from now on, H should be understood as the following function of ρ :

$$H = H(\rho) = \left[\frac{R^2}{\rho^2 + L^2} \right]^3. \tag{3.52}$$

The form of the lagrangian density (3.48) for this ansatz can be straightforwardly computed, with the result:

$$\mathcal{L} = -T_p \rho^{p-1} \sqrt{\tilde{g}} \sqrt{1 + 2Fx' - HF^2}, \quad (3.53)$$

and the equation of motion for the electric field F is:

$$\frac{\partial}{\partial \rho} \left[\frac{\partial \mathcal{L}}{\partial F} \right] = 0. \quad (3.54)$$

This equation can be immediately integrated, namely:

$$\frac{\rho^{p-1} (HF - x')}{\sqrt{1 + 2Fx' - HF^2}} = c, \quad (3.55)$$

where c is a constant. Moreover, from (3.55) we can obtain F as a function of x' and ρ :

$$F = H^{-1} \left[x' + c \frac{\sqrt{H + (x')^2}}{\sqrt{c^2 + \rho^{2(p-1)} H}} \right]. \quad (3.56)$$

Actually, F can be eliminated in a systematic way by means of a Legendre transformation. Indeed, let us define the Routhian density \mathcal{R} as follows:

$$\mathcal{R} = F \frac{\partial \mathcal{L}}{\partial F} - \mathcal{L}. \quad (3.57)$$

By computing the derivative in the explicit expression of \mathcal{L} in (3.53), and by using (3.56), one can readily show that \mathcal{R} can be written as:

$$\mathcal{R} = T_p \sqrt{\tilde{g}} H^{-1} \left[\sqrt{c^2 + \rho^{2(p-1)} H} \sqrt{H + (x')^2} + cx' \right]. \quad (3.58)$$

The equation of motion for x derived from \mathcal{R} is just:

$$\frac{\partial}{\partial \rho} \left[\frac{\partial \mathcal{R}}{\partial x'} \right] = 0. \quad (3.59)$$

A particular solution of this equation can be obtained by requiring the vanishing of $\partial \mathcal{R} / \partial x'$. By computing explicitly this derivative from the expression of \mathcal{R} in (3.58) one easily shows that the value of x' for this particular solution is simply:

$$x' = -\frac{c}{\rho^{p-1}}, \quad (3.60)$$

which, for $p \neq 2$ can be integrated as:

$$x(\rho) = \frac{c}{p-2} \frac{1}{\rho^{p-2}} + \text{constant}, \quad (p \neq 2), \quad (3.61)$$

while for $p = 2$ the Dp-brane has a logarithmic bending of the type $x(\rho) \sim -c \log \rho$. Moreover, after substituting (3.60) on the right-hand side of (3.56) one easily realizes that the worldvolume gauge field F for this configuration vanishes, i.e.:

$$F = 0. \quad (3.62)$$

Actually, it is also easy to verify from (3.56) that the requirement of having vanishing electric gauge field on the worldvolume is equivalent to have a bending given by eq. (3.60). Notice also that the on-shell lagrangian density (3.53) for this configuration becomes $\mathcal{L} = -T_p \rho^{p-1} \sqrt{\tilde{g}}$, which is independent of the distance L . This suggests that the configuration is supersymmetric, a fact that we will verify explicitly in the next subsection by looking at the kappa symmetry of the embedding.

Notice that the embedding (3.60) depends on the constant c . This constant is constrained by a flux quantization condition which, for electric worldvolume gauge fields, was worked out in [45] and reads:

$$\int_{S^{p-1}} \frac{\partial \mathcal{L}}{\partial F} = n T_f, \quad n \in \mathbb{Z}. \quad (3.63)$$

From (3.53) one easily gets:

$$\left. \frac{\partial \mathcal{L}}{\partial F} \right|_{F=0} = T_p \sqrt{\tilde{g}} c, \quad (3.64)$$

which allows one to compute the integral on the left-hand side of (3.63). Let us express the result in terms of the Yang-Mills coupling, which was written in terms of string theory quantities in (2.92). Taking into account that the Dp-brane tension T_p is related to g_{YM} as $T_p = T_f^2 / g_{\text{YM}}^2$, one easily arrives at the following expression of c in terms of the integer n :

$$c = \frac{\alpha' g_{\text{YM}}^2}{\Omega_{p-1}} 2\pi n, \quad (3.65)$$

where Ω_{p-1} is the volume of a unit S^{p-1} , namely $\Omega_{p-1} = 2\pi^{\frac{p}{2}} / \Gamma(\frac{p}{2})$. Physically, the integer n represents the number of fundamental strings that are reconnected to the Dp-brane. Notice that for $p = 3$ eq. (3.65) reduces to $c = n\pi\alpha'g_s$, to be compared with the S-dual relation (3.10).

3.3.1 Supersymmetry

The supersymmetric configurations of a D-brane probe in a given background are those for which the following condition:

$$\Gamma_\kappa \epsilon = \epsilon, \quad (3.66)$$

is satisfied [46]. In eq. (3.66), Γ_κ is a matrix whose explicit expression depends on the embedding of the probe (see below) and ϵ is a Killing spinor of the background. For simplicity we will restrict ourselves to study the kappa symmetry condition (3.66) in the type IIB theory. First of all, let us define the induced worldvolume gamma matrices as:

$$\gamma_m = \partial_m X^M E_M^{\bar{N}} \Gamma_{\bar{N}}, \quad (3.67)$$

where $\Gamma_{\bar{N}}$ are constant ten-dimensional Dirac matrices and $E_M^{\bar{N}}$ is the vielbein for the ten-dimensional metric. Then, if $\gamma_{m_1 m_2 \dots}$ denotes the antisymmetrized product of the induced

gamma matrices (3.67), the kappa symmetry matrix for a Dp-brane in the type IIB theory is [47]:

$$\Gamma_\kappa = \frac{1}{\sqrt{-\det(g + \mathcal{F})}} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} \gamma^{m_1 n_1 \dots m_n n_n} \mathcal{F}_{m_1 n_1} \dots \mathcal{F}_{m_n n_n} \times (\sigma_3)^{\frac{p-3}{2}-n} (i\sigma_2) \Gamma_{(0)} , \quad (3.68)$$

where $\Gamma_{(0)}$ denotes:

$$\Gamma_{(0)} = \frac{1}{(p+1)!} \epsilon^{m_1 \dots m_{p+1}} \gamma_{m_1 \dots m_{p+1}} . \quad (3.69)$$

In eq. (3.68) σ_2 and σ_3 are Pauli matrices that act on the two Majorana-Weyl components (arranged as a two-dimensional vector) of the type IIB spinors.

Let us consider a Dp-brane embedded in the geometry (3.47) according to the ansatz (3.50). Let us assume that we parametrize the S^{p-1} sphere by means of the angles $\alpha^1, \dots, \alpha^{p-1}$. The induced gamma matrices are:

$$\begin{aligned} \gamma_{x^0} &= H^{-\frac{1}{2}} \Gamma_{x^0} , \\ \gamma_\rho &= \Gamma_\rho + H^{-\frac{1}{2}} x' \Gamma_{x^1} , \\ \gamma_{\alpha^1 \dots \alpha^{p-1}} &= \rho^{p-1} \sqrt{\tilde{g}} \Gamma_{\Omega_{p-1}} , \end{aligned} \quad (3.70)$$

where $\Gamma_{\Omega_{p-1}} \equiv \Gamma_{\alpha^1} \dots \Gamma_{\alpha^{p-1}}$. Using these matrices we can write the kappa symmetry matrix Γ_κ in (3.68) as:

$$\Gamma_\kappa = \frac{\rho^{p-1} \sqrt{\tilde{g}}}{\sqrt{-\det(g + \mathcal{F})}} (\sigma_3)^{\frac{p-3}{2}} (i\sigma_2) \left[H^{-\frac{1}{2}} \Gamma_{x^0 \rho} + H^{-1} x' \Gamma_{x^0 x^1} + \mathcal{F} \sigma_3 \right] \Gamma_{\Omega_{p-1}} . \quad (3.71)$$

Let us now study the action of Γ_κ on the Killing spinor ϵ . We shall impose to ϵ the projections corresponding to the Dp-brane and the F1-string, namely:

$$\begin{aligned} (\sigma_3)^{\frac{p-3}{2}} (i\sigma_2) \Gamma_{x^0 \rho} \Gamma_{\Omega_{p-1}} \epsilon &= \epsilon , \\ \sigma_3 \Gamma_{x^0 x^1} \epsilon &= \epsilon . \end{aligned} \quad (3.72)$$

It is now straightforward to verify that

$$\Gamma_\kappa \epsilon = \frac{\rho^{p-1} \sqrt{\tilde{g}}}{\sqrt{-\det(g + \mathcal{F})}} \left[H^{-\frac{1}{2}} + \left(\mathcal{F} + H^{-1} x' \right) (\sigma_3)^{\frac{p-1}{2}} (i\sigma_2) \Gamma_{\Omega_{p-1}} \right] \epsilon . \quad (3.73)$$

We want to impose that the right-hand side of (3.73) be ϵ . It is clear that, if we do not want to impose any further projection to the spinor, we should require that the term that is not proportional to the unit matrix cancels, which happens when:

$$\mathcal{F} + H^{-1} x' = 0 . \quad (3.74)$$

Notice that this condition is equivalent to require the vanishing of F (see eq. (3.51)), as claimed. Moreover, by computing the DBI determinant on the denominator of (3.73) one readily proves that, indeed, eq. (3.66) is satisfied by our configuration.

3.3.2 Fluctuations

Now we will study the fluctuations around the configuration described by eqs. (3.50) and (3.62). We will only analyze the fluctuations on the transverse \vec{z} space, which we will denote by χ . After a straightforward computation, we get that, up to quadratic order, the lagrangian density of these fluctuations is:

$$\mathcal{L} = -\rho^{p-1} \sqrt{\tilde{g}} \left(1 + \frac{c^2}{\rho^{2(p-1)} H} \right) \mathcal{G}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi, \tag{3.75}$$

where the effective metric $\mathcal{G}_{\mu\nu}$ is given by:

$$\mathcal{G}_{\mu\nu} dx^\mu dx^\nu = -H^{-1} (dx^0)^2 + \left(1 + \frac{c^2}{\rho^{2(p-1)} H} \right) (d\rho^2 + \rho^2 d\Omega_{p-1}^2). \tag{3.76}$$

As a check, one can verify that the equation derived from (3.75) for $p = 3$ (i.e. for the F1-D3 intersection) matches precisely that of the transverse scalar fluctuations of the D1-D3 system (i.e. eq. (3.34) with $p = 1$), once the constants c and q are identified. This is, of course, expected from S-duality and implies that the F1-D3 spectrum is continuous and gapless. For $p > 3$ the meson spectrum displays the same characteristics as in the F1-D3 intersection. However, the F1-D2 system behaves differently. Indeed, for $p = 2$ the profile function $x(\rho)$ is logarithmic (see eqs. (3.60) and (3.61)). Moreover, one can check that in this case the effective metric (3.76) in the IR region $\rho \sim 0$ corresponds to an space of the type $\text{Min}_{1,1} \times S^1$. Actually, by studying the fluctuation equation derived from (3.75) for $p = 2$ and $\rho \sim 0$, one can verify that non-oscillatory solutions can exist if the KK momentum in the S^1 is non-zero. As one can check by solving numerically the fluctuation equation, in this case the mass spectrum starts with a finite number of discrete states, followed by a continuum.

4. M2-M5 intersection and codimension one defects in M-theory

We will consider now a close relative in M-theory of the D_p - $D(p+2)$ intersections, namely the M2-M5 intersection along one common spatial dimension. The corresponding array is:

$$\begin{array}{cccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ M2 : & \times & \times & - & - & - & - & - & - & - & - \\ M5 : & \times & - & \times & \times & \times & \times & - & - & - & - \end{array} \tag{4.1}$$

Since this configuration can be somehow thought as the uplift of the D2-D4 intersection to eleven dimensions, we expect a behaviour similar to the one studied in section 3. Indeed, notice that the M5-brane induces a codimension one defect in the M2-brane worldvolume. As in the previous examples we will treat the highest dimensional brane (i.e. the M5-brane) as a probe in the background created by the lower dimensional object, which in this case is the M2-brane. The near-horizon metric of the M2-brane background of eleven-dimensional supergravity is:

$$ds^2 = \frac{r^4}{R^4} dx_{1,2}^2 + \frac{R^2}{r^2} d\vec{r}^2, \tag{4.2}$$

where R is constant, $dx_{1,2}^2$ represents the Minkowski metric in the directions x^0, x^1, x^2 of the M2-brane worldvolume and \vec{r} is an eight-dimensional vector transverse to the M2-brane. The metric (4.2) is the one of the $AdS_4 \times S^7$ space, where the radius of the AdS_4 (S^7) factor is $R/2$ (R). The actual value of R for a stack of N coincident M2-branes is:

$$R^6 = 32\pi^2 l_p^6 N, \tag{4.3}$$

where l_p is the Planck length in eleven dimensions. This background is also endowed with a three-form potential $C^{(3)}$, whose explicit expression is:

$$C^{(3)} = \frac{r^6}{R^6} dx^0 \wedge dx^1 \wedge dx^2. \tag{4.4}$$

The dynamics of the M5-brane probe is governed by the so-called PST action [48]. In the PST formalism the worldvolume fields are a three-form field strength F and an auxiliary scalar a . This action is given by [48]:

$$S = T_{M5} \int d^6 \xi \left[-\sqrt{-\det(g_{ij} + \tilde{H}_{ij})} + \frac{\sqrt{-\det g}}{4\partial a \cdot \partial a} \partial_i a (\star H)^{ijk} H_{jkl} \partial^l a \right] + T_{M5} \int \left[P[C^{(6)}] + \frac{1}{2} F \wedge P[C^{(3)}] \right], \tag{4.5}$$

where $T_{M5} = 1/(2\pi)^5 l_p^6$ is the tension of the M5-brane, g is the induced metric and H is the following combination of the worldvolume gauge field F and the pullback of the three-form $C^{(3)}$:

$$H = F - P[C^{(3)}]. \tag{4.6}$$

Moreover, the field \tilde{H} is defined as follows:

$$\tilde{H}^{ij} = \frac{1}{3! \sqrt{-\det g}} \frac{1}{\sqrt{-(\partial a)^2}} \epsilon^{ijklmn} \partial_k a H_{lmn}, \tag{4.7}$$

and the worldvolume indices in (4.5) are lowered with the induced metric g_{ij} .

In order to study the embedding of the M5-brane in the M2-brane background, let us introduce a more convenient set of coordinates. Let us split the vector \vec{r} as $\vec{r} = (\vec{y}, \vec{z})$, where $\vec{y} = (y^1, \dots, y^4)$ is the position vector along the directions 3456 in the array (4.1) and $\vec{z} = (z^1, \dots, z^4)$ corresponds to the directions 7, 8, 9 and 10. Obviously, if $\rho^2 = \vec{y} \cdot \vec{y}$, one has that $\vec{r}^2 = \rho^2 + \vec{z}^2$ and $d\vec{r}^2 = d\rho^2 + \rho^2 d\Omega_3^2 + d\vec{z}^2$, where $d\Omega_3^2$ is the line element of a three-sphere. Thus, the metric (4.2) becomes:

$$ds^2 = \frac{(\rho^2 + \vec{z}^2)^2}{R^4} dx_{1,2}^2 + \frac{R^2}{\rho^2 + \vec{z}^2} (d\rho^2 + \rho^2 d\Omega_3^2 + d\vec{z}^2). \tag{4.8}$$

We will now choose x^0, x^1, ρ and the three angular coordinates that parametrize $d\Omega_3^2$ as our worldvolume coordinates ξ^i . Moreover, we will assume that the vector \vec{z} is constant and we will denote its modulus by L , namely:

$$|\vec{z}| = L. \tag{4.9}$$

To specify completely the embedding of the M5-brane we must give the form of the remaining scalar x^2 as a function of the worldvolume coordinates. For simplicity we will assume that x^2 only depends on the radial coordinate ρ , i.e. that:

$$x^2 = x(\rho) . \tag{4.10}$$

Moreover, we will switch on a magnetic field F along the three-sphere of the M5-brane worldvolume, in the form:

$$F = q \text{Vol}(S^3) , \tag{4.11}$$

where q is a constant and $\text{Vol}(S^3)$ is the volume form of the worldvolume three-sphere. Notice that the induced metric for this configuration is given by:

$$g_{ij}d\xi^i d\xi^j = \frac{(\rho^2 + L^2)^2}{R^4} dx_{1,1}^2 + \frac{R^2}{\rho^2 + L^2} \left\{ \left(1 + \frac{(\rho^2 + L^2)^3}{R^6} (x')^2 \right) d\rho^2 + \rho^2 d\Omega_3^2 \right\} . \tag{4.12}$$

In order to write the PST action for our ansatz we must specify the value of the PST scalar a . As pointed out in ref. [48] the field a can be eliminated by gauge fixing, at the expense of losing manifest covariance. Here we will choose a gauge such that the auxiliary PST scalar is:

$$a = x_1 . \tag{4.13}$$

It is now straightforward to prove that the only non-vanishing component of the field \tilde{H} is:

$$\tilde{H}_{x^0\rho} = -\frac{i}{R^4} \frac{(\rho^2 + L^2)^2}{\rho^3} \left(1 + \frac{(\rho^2 + L^2)^3}{R^6} (x')^2 \right)^{\frac{1}{2}} q . \tag{4.14}$$

Using these results we can write the PST action (4.5) as:

$$S = -2\pi^2 T_{M5} \int d^2x d\rho \left[\rho^3 \sqrt{1 + \frac{(\rho^2 + L^2)^3}{R^6} (x')^2} \sqrt{1 + \frac{(\rho^2 + L^2)^3}{R^6} \frac{q^2}{\rho^6}} + \frac{(\rho^2 + L^2)^3}{R^6} q x' \right] . \tag{4.15}$$

Let \mathcal{L} be the lagrangian density for the PST action, which we can take as given by the expression inside the brackets in (4.15). Since x does not appear explicitly in the action, one can immediately write a first integral of the equation of motion of $x(\rho)$, namely:

$$\frac{\partial \mathcal{L}}{\partial x'} = \text{constant} . \tag{4.16}$$

By setting the constant on the right-hand side of (4.16) equal to zero, this equation reduces to a simple first-order equation for $x(\rho)$, i.e.:

$$x' = -\frac{q}{\rho^3} , \tag{4.17}$$

which can be immediately integrated to give:

$$x(\rho) = \bar{x} + \frac{q}{2\rho^2}, \tag{4.18}$$

where \bar{x} is a constant. Notice that the flux parametrized by q induces a bending of the M5-brane, which is characterized by the non-trivial dependence of x on the holographic coordinate ρ . Actually, when the first-order eq. (4.17) holds, the two square roots in (4.15) are equal and there is a cancellation with the last term in (4.15). Indeed, the on-shell action takes the form:

$$S = -2\pi^2 T_5 \int d^2x d\rho \rho^3, \tag{4.19}$$

which is independent of the M2-M5 distance L . This is usually a signal of supersymmetry and, indeed, we will verify in appendix B that the embeddings in which the flux and the bending are related as in (4.17) are kappa symmetric. Thus, eq. (4.17) can be regarded as the first-order BPS equation required by supersymmetry. Notice also that the three-form flux (4.11) induces M2-brane charge in the M5-brane worldvolume, as it is manifest from the form of the PST action (4.5). In complete analogy with the Dp-D(p+2) system, we can interpret the present M-theory configuration in terms of M2-branes that recombine with the M5-brane. Moreover, in order to gain further insight on the effect of the bending, let us rewrite the induced metric (4.12) when the explicit form of $x(\rho)$ written in eq. (4.18) is taken into account. One gets:

$$\frac{(\rho^2 + L^2)^2}{R^4} dx_{1,1}^2 + \frac{R^2}{\rho^2 + L^2} \left\{ \left(1 + \frac{q^2}{R^6} \frac{(\rho^2 + L^2)^3}{\rho^6} \right) d\rho^2 + \rho^2 d\Omega_3^2 \right\}. \tag{4.20}$$

From (4.20) one readily notices that the UV induced metric at $\rho \rightarrow \infty$ (or, equivalently when the M2-M5 distance L is zero) takes the form $AdS_3(R_{\text{eff}}/2) \times S^3(R)$, where the AdS_3 radius R_{eff} depends on the flux as:

$$R_{\text{eff}} = \left(1 + \frac{q^2}{R^6} \right)^{\frac{1}{2}} R. \tag{4.21}$$

Therefore, our M5-brane is wrapping an AdS_3 submanifold of the AdS_4 background. Actually, there are infinite ways of embedding an AdS_3 within an AdS_4 space and the bending of the probe induced by the flux is selecting one particular case of these embeddings. In order to shed light on this, let us suppose that we have an AdS_4 metric of the form:

$$ds_{AdS_4}^2 = \frac{\rho^4}{R^4} dx_{1,2}^2 + \frac{R^2}{\rho^2} d\rho^2. \tag{4.22}$$

Let us now change variables from $(x^{0,1}, x^2, \rho)$ to $(\hat{x}^{0,1}, \varrho, \eta)$, as follows:

$$x^{0,1} = 2\hat{x}^{0,1}, \quad x^2 = \bar{x} + \frac{2}{\varrho} \tanh \eta, \quad \rho^2 = \frac{R^3}{4} \varrho \cosh \eta, \tag{4.23}$$

where \bar{x} is a constant. In these new variables the AdS_4 metric (4.22) can be written as a foliation by AdS_3 slices, namely:

$$ds_{AdS_4}^2 = \frac{R^2}{4} (\cosh^2 \eta ds_{AdS_3}^2 + d\eta^2), \tag{4.24}$$

where $ds_{AdS_3}^2$ is given by:

$$ds_{AdS_3}^2 = \varrho^2 (-(d\hat{x}^0)^2 + (d\hat{x}^1)^2) + \frac{d\varrho^2}{\varrho^2}. \quad (4.25)$$

Clearly the AdS_3 slices in (4.24) can be obtained by taking $\eta = \text{constant}$. The radius of such AdS_3 slice is $R_{\text{eff}}/2$, with:

$$R_{\text{eff}} = R \cosh \eta. \quad (4.26)$$

Moreover, one can verify easily by using the change of variables (4.23) that our embedding (4.18) corresponds to one of such AdS_3 slices with:

$$\eta = \eta_q = \sinh^{-1} \left(\frac{q}{R^3} \right). \quad (4.27)$$

Furthermore, one can check that the AdS_3 radius R_{eff} of eq. (4.26) reduces to (4.21) when $\eta = \eta_q$.

4.1 Fluctuations

Let us now study the fluctuations of the M2-M5 intersection. For simplicity we will focus on the fluctuations of the transverse scalars which, without loss of generality, we will parametrize as:

$$z^1 = L + \chi^1, \quad z^m = \chi^m, \quad (m = 2, \dots, 4). \quad (4.28)$$

Let us substitute this ansatz in the PST action and keep up to second order terms in the fluctuation χ . As the calculation is very similar to the one performed in subsection 3.2, we skip the details and give the final result for the effective lagrangian of the fluctuations, namely:

$$\mathcal{L} = -\rho^3 \sqrt{\tilde{g}} \frac{R^2}{\rho^2 + L^2} \left[1 + \frac{q^2}{R^6} \frac{(\rho^2 + L^2)^3}{\rho^6} \right] \hat{G}^{ij} \partial_i \chi \partial_j \chi, \quad (4.29)$$

where \tilde{g} is the determinant of the round metric of the S^3 and \hat{G}_{ij} is the following effective metric on the M5-brane worldvolume:

$$\hat{G}_{ij} d\xi^i d\xi^j = \frac{(\rho^2 + L^2)^2}{R^4} dx_{1,1}^2 + \frac{R^2}{\rho^2 + L^2} \left(1 + \frac{q^2}{R^6} \frac{(\rho^2 + L^2)^3}{\rho^6} \right) (d\rho^2 + \rho^2 d\Omega_3^2). \quad (4.30)$$

Notice the close analogy with the Dp-D(p+2) system studied in subsection 3.2. Actually (4.30) is the analogue of the open string metric in this case. The equation of motion for the scalars can be derived straightforwardly from the lagrangian density (4.29). For $q = 0$ this equation was integrated in ref. [20], where the meson mass spectra was also computed. This fluxless spectra is discrete and displays a mass gap. As happened with the codimension one defects in type II theory studied in section 3, the situation changes drastically when $q \neq 0$. To verify this fact let us study the form of the effective metric (4.30) in the UV ($\rho \rightarrow \infty$) and in the IR ($\rho \rightarrow 0$). After studying this metric when $\rho \rightarrow \infty$, one easily concludes that the UV is of the form $AdS_3(R_{\text{eff}}/2) \times S^3(R_{\text{eff}})$, where R_{eff} is just the

effective radius with flux written in (4.21). Thus, the effect of the flux in the UV is just a redefinition of the AdS_3 and S^3 radii of the metric governing the fluctuations. On the contrary, for $q \neq 0$ the behaviour of this metric in the IR changes drastically with respect to the fluxless case. Indeed, for $\rho \approx 0$ the metric (4.30) takes the form:

$$\frac{L^4}{R^4} \left[dx_{1,1}^2 + q^2 \left(\frac{d\rho^2}{\rho^6} + \frac{1}{\rho^4} d\Omega_2^2 \right) \right], \quad (\rho \approx 0). \quad (4.31)$$

Notice the analogy of (4.31) with the IR metric (3.42) of the Dp-D(p+2) defects. Actually, the IR limit of the equation of motion of the fluctuation can be integrated, as in (3.39), in terms of Bessel functions, which for $\rho \approx 0$ behave as plane waves of the form $e^{\pm iMx}$, where x is the function (4.18). Notice that $\rho \approx 0$ corresponds to large x in (4.18). Thus, the fluctuations spread out of the defect and oscillate infinitely at the IR and, as a consequence, the mass spectrum is continuous and gapless. In complete analogy with the Dp-D(p+2) with flux, this is a consequence of the recombination of the M2- and M5-branes and should be understood microscopically in terms of dielectric multiple M2-branes polarized into a M5-brane, once such an action is constructed.

5. The codimension two defect

We now analyze the codimension two defect, which can be engineered in type II string theory as a Dp-Dp intersection over $p - 2$ spatial dimensions. We will consider a single Dp'-brane intersecting a stack of N Dp-branes, according to the array:

$$\begin{array}{cccccccc} 1 & \cdots & p-2 & p-1 & p & p+1 & p+2 & \cdots & 9 \\ Dp : & \times & \cdots & \times & \times & \times & - & - & \cdots & - \\ Dp' : & \times & \cdots & \times & - & - & \times & \times & \cdots & - \end{array} \quad (5.1)$$

In the limit of large N we can think of the system as a probe Dp'-brane in the near horizon geometry of the Dp-branes given by (2.60), (2.62) and (2.63). It is clear from the array (5.1) that the Dp'-brane produces a defect of codimension two in the field theory dual to the stack of Dp-branes. The defect field theory dual to the D3-D3 intersection was studied in detail in ref. [28] (see also ref. [30]). Notice also that this same D3-D3 intersection was considered in [49] in connection with the surface operators of $\mathcal{N} = 4$ super Yang-Mills theory, in the context of the geometric Langlands program.

In order to describe the dynamics of the Dp'-brane probe, let us relabel the x^{p-1} and x^p coordinates appearing in the metric (2.60) as:

$$\lambda^1 \equiv x^{p-1}, \quad \lambda^2 \equiv x^p. \quad (5.2)$$

Moreover, we will split the coordinates \vec{r} transverse to the Dp-branes as $\vec{r} = (\vec{y}, \vec{z})$, where $\vec{y} = (y^1, y^2)$ corresponds to the p+1 and p+2 directions in (5.1) and $\vec{z} = (z^1, \dots, z^{7-p})$ to the remaining transverse coordinates. With this split of coordinates the background metric reads:

$$ds^2 = \left[\frac{\vec{y}^2 + \vec{z}^2}{R^2} \right]^\alpha (dx_{1,p-2}^2 + d\vec{\lambda}^2) + \left[\frac{R^2}{\vec{y}^2 + \vec{z}^2} \right]^\alpha (d\vec{y}^2 + d\vec{z}^2), \quad (5.3)$$

where $dx_{1,p-2}^2$ is the Minkowski metric in the coordinates x^0, \dots, x^{p-2} and α has been defined in (2.60).

5.1 Supersymmetric embeddings

To study the embeddings of the Dp' -brane probe in the background (2.60)–(2.63) let us consider $\xi^m = (x^0, \dots, x^{p-2}, y^1, y^2)$ as worldvolume coordinates. In this approach $\vec{\lambda}$ and \vec{z} are scalar fields that characterize the embedding. Actually, we will restrict ourselves to the case in which $\vec{\lambda}$ depends only on the \vec{y} coordinates (i.e. $\vec{\lambda} = \vec{\lambda}(\vec{y})$) and the transverse separation $|\vec{z}|$ is constant, i.e. $|\vec{z}| = L$.

In order to characterize the embeddings of the probe that preserve supersymmetry, let us try to implement the kappa symmetry condition (3.66). The induced gamma matrices γ_{x^μ} ($\mu = 0, \dots, p-2$) and γ_{y^i} ($i = 1, 2$) can be computed from eq. (3.67), with the result:

$$\begin{aligned} \gamma_{x^\mu} &= \left[\frac{\rho^2 + L^2}{R^2} \right]^{\frac{\alpha}{2}} \Gamma_{x^\mu} , \\ \gamma_{y^i} &= \left[\frac{R^2}{\rho^2 + L^2} \right]^{\frac{\alpha}{2}} \Gamma_{y^i} + \left[\frac{\rho^2 + L^2}{R^2} \right]^{\frac{\alpha}{2}} \left[\partial_i \lambda^1 \Gamma_{\lambda^1} + \partial_i \lambda^2 \Gamma_{\lambda^2} \right] , \end{aligned} \quad (5.4)$$

where $\partial_i \equiv \partial_{y^i}$ and, as before, we have defined $\rho^2 = \vec{y} \cdot \vec{y}$. To simplify matters, let us assume that p is odd and, thus, we are working on the type IIB theory. The general expression of the kappa symmetry matrix Γ_κ has been written in eq. (3.68). For the present case this matrix reads:

$$\Gamma_\kappa = \frac{1}{\sqrt{-\det(g)}} \left[\frac{\rho^2 + L^2}{R^2} \right]^{\frac{(p-1)\alpha}{2}} (\sigma_3)^{\frac{p-3}{2}} (i\sigma_2) \Gamma_{x^0 \dots x^{p-2}} \gamma_{y^1 y^2} . \quad (5.5)$$

The antisymmetrized product $\gamma_{y^1 y^2}$ can be straightforwardly computed from the expression of the γ_{y^i} matrices in (5.4). One gets:

$$\begin{aligned} \left[\frac{\rho^2 + L^2}{R^2} \right]^\alpha \gamma_{y^1 y^2} &= \Gamma_{y^1 y^2} + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \left(\partial_1 \lambda^1 \partial_2 \lambda^2 - \partial_1 \lambda^2 \partial_2 \lambda^1 \right) \Gamma_{\lambda^1 \lambda^2} \\ &+ \left[\frac{\rho^2 + L^2}{R^2} \right]^\alpha \left[\partial_2 \lambda^1 \Gamma_{y^1 \lambda^1} + \partial_2 \lambda^2 \Gamma_{y^1 \lambda^2} - \partial_1 \lambda^1 \Gamma_{y^2 \lambda^1} - \partial_1 \lambda^2 \Gamma_{y^2 \lambda^2} \right] . \end{aligned} \quad (5.6)$$

Let us now use this expression to fulfill the condition $\Gamma_\kappa \epsilon = \epsilon$, where ϵ is a Killing spinor of the Dp -brane background. For a generic value of p these Dp -brane spinors satisfy the projection condition:

$$(\sigma_3)^{\frac{p-3}{2}} (i\sigma_2) \Gamma_{x^0 \dots x^{p-2}} \Gamma_{\lambda^1 \lambda^2} \epsilon = \epsilon . \quad (5.7)$$

Moreover we will also impose the projection corresponding to the Dp' -brane probe, namely:

$$(\sigma_3)^{\frac{p-3}{2}} (i\sigma_2) \Gamma_{x^0 \dots x^{p-2}} \Gamma_{y^1 y^2} \epsilon = \epsilon . \quad (5.8)$$

Notice that (5.7) and (5.8) are compatible, as it should for a supersymmetric intersection. Moreover, they can be combined to give:

$$\Gamma_{y^1 y^2} \epsilon = \Gamma_{\lambda^1 \lambda^2} \epsilon , \tag{5.9}$$

which implies that:

$$\begin{aligned} \left[\frac{\rho^2 + L^2}{R^2} \right]^\alpha \gamma_{y^1 y^2} \epsilon &= \left[1 + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \left(\partial_1 \lambda^1 \partial_2 \lambda^2 - \partial_1 \lambda^2 \partial_2 \lambda^1 \right) \right] \Gamma_{\lambda^1 \lambda^2} \epsilon \\ &+ \left[\frac{\rho^2 + L^2}{R^2} \right]^\alpha \left[(\partial_2 \lambda^1 + \partial_1 \lambda^2) \Gamma_{y^1 \lambda^1} + (\partial_2 \lambda^2 - \partial_1 \lambda^1) \Gamma_{y^1 \lambda^2} \right] \epsilon . \end{aligned} \tag{5.10}$$

We can now use this result to compute $\Gamma_\kappa \epsilon$, where Γ_κ is given in (5.5). By using the condition (5.9) one easily gets that the terms of the first line of the right-hand side of (5.10) give contributions proportional to the identity matrix, while those on the second line of (5.10) give rise to terms that contain matrices that do not act on ϵ as the identity unless we impose some extra projections which would reduce the amount of preserved supersymmetry. Since we do not want this to happen, we require that the coefficients of $\Gamma_{y^1 \lambda^1}$ and $\Gamma_{y^1 \lambda^2}$ in (5.10) vanish, i.e.:

$$\partial_1 \lambda^1 = \partial_2 \lambda^2 , \quad \partial_2 \lambda^1 = -\partial_1 \lambda^2 . \tag{5.11}$$

Notice that eq. (5.11) is nothing but the Cauchy-Riemann equations. Indeed, let us define the following complex combinations of worldvolume coordinates and scalars ³:

$$Z = y^1 + iy^2 , \quad W = \lambda^1 + i\lambda^2 . \tag{5.12}$$

In addition, if we define the holomorphic and antiholomorphic derivatives as:

$$\partial = \frac{1}{2} (\partial_1 - i\partial_2) , \quad \bar{\partial} = \frac{1}{2} (\partial_1 + i\partial_2) , \tag{5.13}$$

then (5.11) can be written as:

$$\bar{\partial} W = 0 , \tag{5.14}$$

whose general solution is an arbitrary holomorphic function of Z , namely:

$$W = W(Z) . \tag{5.15}$$

It is also straightforward to check that for these holomorphic embeddings the induced metric takes the form:

$$\left[\frac{\rho^2 + L^2}{R^2} \right]^\alpha dx_{1,p-2}^2 + \left[\frac{R^2}{\rho^2 + L^2} \right]^\alpha \left[1 + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \partial W \bar{\partial} \bar{W} \right] dZ d\bar{Z} , \tag{5.16}$$

³The complex worldvolume coordinate Z should not be confused with the real transverse scalars \vec{z} . Notice also that $\rho^2 = |Z|^2$.

whose determinant is:

$$\sqrt{-\det(g)} = \left[\frac{\rho^2 + L^2}{R^2} \right]^{\frac{(p-3)\alpha}{2}} \left[1 + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \partial W \bar{\partial} \bar{W} \right]. \quad (5.17)$$

Using this result one can easily verify that the condition $\Gamma_\kappa \epsilon = \epsilon$ is indeed satisfied. Moreover, for these holomorphic embeddings the DBI lagrangian density takes the form:

$$\mathcal{L}_{\text{DBI}} = -T_p e^{-\phi} \sqrt{-\det(g)} = -T_p \left[1 + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \partial W \bar{\partial} \bar{W} \right], \quad (5.18)$$

where we have used the value of $e^{-\phi}$ for the Dp-brane background displayed in eq. (2.62). On the other hand, from the form of the RR potential $C^{(p+1)}$ written in (2.63) one can readily check that, for these holomorphic embeddings, the WZ piece of the lagrangian can be written as:

$$\mathcal{L}_{\text{WZ}} = T_p \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \partial W \bar{\partial} \bar{W}. \quad (5.19)$$

Notice that, for these holomorphic embeddings, the WZ lagrangian \mathcal{L}_{WZ} cancels against the second term of \mathcal{L}_{DBI} (see eq. (5.18)). Thus, once again, the on-shell action is independent of the distance L , a result which is a consequence of supersymmetry and holomorphicity.

Notice that, from the point of view of supersymmetry, any holomorphic curve $W(Z)$ is allowed. Obviously, we could have $W = \text{constant}$. In this case the probe sits at a particular constant point of its transverse space and does not recombine with branes of the background. If, on the contrary, $W(Z)$ is not constant, Liouville theorem ensures us that it cannot be bounded in the whole complex plane. The points at which $|W|$ diverge are spikes of the probe profile, and one can interpret them as the points where the probe and background branes merge. Notice that, as opposed to the other cases studied in this paper, the non-trivial profile of the embedding is not induced by the addition of any worldvolume field. Thus, we are not dissolving any further charge in the probe brane and a dielectric interpretation is not possible now.

The field theory dual for the $p = 3$ system has been worked out in refs. [28] and [30]. The dual gauge theory for this D3-D3 intersection was shown to correspond to two $\mathcal{N} = 4$ four-dimensional theories coupled to each other through a two-dimensional defect that hosts a bifundamental hypermultiplet. The Coulomb branch corresponds to taking the embedding $W = \text{constant}$. Moreover, one can seek for a Higgs branch arising from the corresponding D and F flatness conditions of the supersymmetric defect theory. Actually, it was shown in [28, 30] that this Higgs branch corresponds to the embedding $W = c/Z$, where c is a constant. Interestingly, only for these embeddings the induced UV metric is of the form $AdS_3 \times S^1$. Indeed, one can check that the metric (5.16) for $p = 3$ (and $\alpha = 1$) and for the profile $W = c/Z$ reduces in the UV to that of the $AdS_3 \times S^1$ space, where the two factors have the same radii $R_{\text{eff}} = \sqrt{1 + \frac{c^2}{R^4}} R$. Thus, as in the M2-M5 intersection of section 4, the constant c parametrizes the particular $AdS_3 \times S^1$ slice of the $AdS_5 \times S^5$ space that is occupied by our D3-brane probe.

5.2 Fluctuations of the Dp-Dp intersection

Let us now study the fluctuations around the previous configurations. We will concentrate on the fluctuations of the scalars transverse to both types of branes, i.e. those along the \vec{z} directions. Let χ be one of such fields. Expanding the action up to quadratic order in the fluctuations it is easy to see that the lagrangian density for χ is:

$$\mathcal{L} = - \left[\frac{R^2}{\rho^2 + L^2} \right]^\alpha \left[1 + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \partial W \bar{\partial} \bar{W} \right] \mathcal{G}^{mn} \partial_m \chi \partial_n \chi, \quad (5.20)$$

where \mathcal{G}_{mn} is the induced metric (5.16). Let us parametrize the complex variable Z in terms of polar coordinates as $Z = \rho e^{i\theta}$ and let us separate variables in the fluctuation equation as

$$\chi = e^{ikx} e^{il\theta} \xi(\rho), \quad (5.21)$$

where the product kx is performed with the Minkowski metric of the defect. If $M^2 = -k^2$, the equation of motion for the radial function $\xi(\rho)$ takes the form:

$$\left[\left[\frac{R^2}{\rho^2 + L^2} \right]^{2\alpha} \left[1 + \left[\frac{\rho^2 + L^2}{R^2} \right]^{2\alpha} \partial W \bar{\partial} \bar{W} \right] M^2 - \frac{l^2}{\rho^2} + \frac{1}{\rho} \partial_\rho (\rho \partial_\rho) \right] \xi(\rho) = 0. \quad (5.22)$$

For $W = \text{constant}$, eq. (5.22) was solved in ref. [20], where it was shown to give rise to a mass gap and a discrete spectrum of M . As in the case of the codimension one defects, this conclusion changes completely when we go to the Higgs branch. Indeed, let us consider the embeddings with $W \sim 1/Z$. One can readily prove that for $\rho \rightarrow \infty$ the function $\xi(\rho)$ behaves as $\xi(\rho) \sim c_1 \rho^l + c_2 \rho^{-l}$, which is exactly the same behaviour as in the $W = \text{constant}$ case. However, in the opposite limit $\rho \rightarrow 0$ the fluctuation equation can be solved in terms of Bessel functions which oscillate infinitely as $\rho \rightarrow 0$. Notice that, for our Higgs branch embeddings, $\rho \rightarrow 0$ means $W \rightarrow \infty$ and, therefore, the fluctuations are no longer localized at the defect, as it happened in the case of the Dp-D(p+2) and M2-M5 intersections. Thus we conclude that, also in this case, the mass gap is lost and the spectrum is continuous.

6. Conclusions

In this paper we have studied the holographic description of the Higgs branch of a large class of theories with fundamental matter. These theories are embedded in string theory as supersymmetric systems of intersecting branes. The strings joining both kind of branes give rise to bifundamental matter confined to the intersection, which once the suitable field theory limit is taken, becomes fundamental matter with a flavor symmetry.

The general picture that emerges from our results is that the Higgs phase is realized by recombining both types of intersecting branes. From the point of view of the higher dimensional flavor brane the recombination takes place when a suitable embedding is chosen and/or some flux of the worldvolume gauge field is switched on. This flux is dissolving color brane charge in the flavor branes and, thus, it is tempting to search for a microscopical description from the point of view of those dissolved branes. Indeed, we have seen that the

vacuum conditions of the dielectric description (when this description is available) match exactly the F- and D-flatness constraints that give rise to the Higgs phase on the field theory side, which gives support to our holographic description of the Higgs branch.

The first case studied was the Dp-D(p+4) intersections, where the flavor D(p+4)-branes fill completely the worldvolume directions of the color Dp-brane. Following [26], we argued that the holographic description of the Higgs branch of this system corresponds to having a self-dual gauge field along the directions of the worldvolume of the D(p+4)-brane that are orthogonal to the Dp-brane. To confirm this statement we have worked out in detail the microscopic description of this system and we have computed the meson mass spectra as a function of the quark VEV.

We also analyzed other intersections that are dual to gauge theories containing defects of non-vanishing codimension. The paradigmatic example of these theories is the Dp-D(p+2) system, where a detailed microscopic description can be found. Other cases include the M2-M5 intersection in M-theory as well as the Dp-Dp system, which gives rise to a codimension two defect. In this latter case the field theory limit does not decouple the flavor symmetry, so we actually have a $SU(N) \times SU(M)$ theory. In addition, the profile of the intersection is only constrained to be holomorphic in certain coordinates, but is otherwise unspecified. In any case, it turns out that conformal invariance in the UV is preserved only for two particular curves, which can be shown to correspond to the Coulomb and Higgs phases (see [28]). In all these non-zero codimension defect theories we studied the meson spectrum and we have shown that it is continuous and that the mass gap is lost. The reason behind this result is the fact that, due to the recombination of color and flavor branes in the Higgs branch, the defect can spread over the whole bulk, which leads to an effective Minkowski worldvolume metric in the IR for the flavor brane. This implies the loss of a KK scale coming from a compact manifold and, therefore, the disappearance of the discrete spectrum. Notice that the case of the Dp-D(p+4) system is different, since in this case the defect fills the whole color brane and there is no room for spreading on the Higgs branch.

Also the Dp-Dp case deserves special attention, since it behaves in a completely different manner to all the other intersections. As we already mentioned the intersection profile is not uniquely fixed by supersymmetry. However, just for two of all the possible embeddings we recover conformal invariance in the UV. While one of them corresponds to the Coulomb phase, the other corresponds to the Higgs phase. It should be stressed that in this case there is no need for extra flux to get the Higgs phase, which in this sense is purely geometrical. The other important difference is that in this case the field theory limit does not decouple any of the gauge symmetries. Then, our fields will be bifundamentals under the gauge group on each Dp-brane. Taking into account the relation with the surface operators in gauge theories [49], it would be interesting to gain more understanding of this system.

Let us now discuss some of the possible extensions of our work. Notice that our analysis has been performed in the probe approximation, in which we neglect the backreaction of the flavor branes on the geometry. This approximation is valid when the number of flavor branes is small as compared to the number of color branes. The analysis of the backreacted

geometry corresponding to the Higgs branch is of obvious interest. In particular it would be very exciting to find the way in which the backreacted geometry encodes some of the phenomena that we have uncovered in the probe approximation. Actually, the backreacted geometry corresponding to the D3-D5 intersection was found in refs. [51, 52]. Also, it would be interesting to see if one can apply the smearing procedure proposed in [53] (see also [54]) to find a solution of the equations of motion of the gravity plus branes systems studied in this paper.

Another problem of great interest is trying to describe holographically (even in the probe approximation) the Higgs branch of theories with less supersymmetry. The most obvious case to look at would be that of branes intersecting on the conifold, such as the D3-D7 systems in the Klebanov-Witten model [55] and its generalizations. Actually, the supersymmetric D3-D5 intersections with flux on the conifold and on more general Sasaki-Einstein cones were obtained in ref. [16, 56]. These configurations are the analogue of the ones analyzed in section 3, and it would be desirable to find its field theory interpretation.

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A. Fluctuations of the Dp-D(p+2) intersection

In this appendix we will complete the analysis of the fluctuations of the Dp-D(p+2) system of section 3. Recall that the Lagrangian that governs these fluctuations has been written in eq. (3.32). In section 3 we already studied the equation of motion of the transverse scalars χ and we concluded that the associated meson mass spectrum is continuous and gapless. The other fields in (3.32) are the scalar x (which is transverse to the D(p+2)-brane and is directed along the p^{th} direction of the Dp-brane worldvolume) and the gauge field f_{ab} . The equation of motion of x reads:

$$\partial_a \left[\frac{\rho^2}{h} \sqrt{\tilde{g}} \hat{G}^{ab} \partial_b x \right] - \frac{C}{2} \epsilon^{ij} f_{ij} = 0, \tag{A.1}$$

while that of the gauge field is:

$$\partial_a \left[\rho^2 \sqrt{\tilde{g}} \left(1 + \frac{q^2}{\rho^4 h^2} \right) f^{ab} \right] - C \epsilon^{bi} \partial_i x = 0, \tag{A.2}$$

where h and C are the functions of ρ defined in eqs. (2.66) and (3.33) and ϵ^{bi} is zero unless b is an index along the two-sphere. Notice that eqs. (A.1) and (A.2) are coupled. Let us decouple them by using the same method as that applied in [20] for the $q = 0$ case. As in ref. [20] we define the following two types of vector spherical harmonics for the two-sphere:

$$Y_i^l(S^2) \equiv \nabla_i Y^l(S^2), \quad \hat{Y}_i^l(S^2) \equiv \frac{1}{\sqrt{g}} \tilde{g}_{ij} \epsilon^{jk} \nabla_k Y^l(S^2), \quad (\text{A.3})$$

where $Y^l(S^2)$ is a scalar harmonic on S^2 . We now study the different types of modes following closely the analysis of ref. [20].

A.1 Type I modes

The type I modes are the ones that involve the scalar field x and the components of the gauge field strength along the S^2 directions. The ansatz that we will adopt for x is the following:

$$x = \Lambda(x^\mu, \rho) Y^l(S^2). \quad (\text{A.4})$$

Moreover, if a_a denote the components of the gauge field potential for f_{ab} , we will take:

$$a_\mu = 0, \quad a_\rho = 0, \quad a_i = \phi(x^\mu, \rho) \hat{Y}_i^l(S^2). \quad (\text{A.5})$$

Using this ansatz, the equation for x becomes:

$$\rho^2 \partial^\mu \partial_\mu \Lambda + \partial_\rho \left[\frac{\rho^6}{q^2 + \rho^4 h^2} \partial_\rho \Lambda \right] - l(l+1) \frac{\rho^4}{q^2 + \rho^4 h^2} \Lambda - C l(l+1) \phi = 0. \quad (\text{A.6})$$

Then, by using the property $\nabla^i \hat{Y}_i^l = 0$, which follows directly from the definition (A.3), one can check that (A.2) reduces to:

$$\partial^\mu \partial_\mu \phi + \partial_\rho \left[\frac{\rho^4}{q^2 + \rho^4 h^2} \partial_\rho \phi \right] - l(l+1) \frac{\rho^2}{q^2 + \rho^4 h^2} \phi - C \Lambda = 0. \quad (\text{A.7})$$

Let us now define the function $V(x^\mu, \rho)$ as:

$$V = \rho \Lambda, \quad (\text{A.8})$$

and the following second-order differential operator \mathcal{O}^2 :

$$\mathcal{O}^2 \psi \equiv \frac{1}{C} \left[\partial^\mu \partial_\mu \psi + \partial_\rho \left(\frac{\rho^4}{q^2 + \rho^4 h^2} \partial_\rho \psi \right) \right]. \quad (\text{A.9})$$

Then, the equations of V and ϕ can be written as the following system of differential equations:

$$\begin{aligned} \mathcal{O}^2 V &= \left[\frac{1}{\rho} + l(l+1) \frac{\rho^2}{(q^2 + \rho^4 h^2) C} \right] V + \frac{l(l+1)}{\rho} \phi, \\ \mathcal{O}^2 \phi &= l(l+1) \frac{\rho^2}{(q^2 + \rho^4 h^2) C} \phi + \frac{1}{\rho} V. \end{aligned} \quad (\text{A.10})$$

In order to decouple this system, let us define the functions Z^\pm as:

$$Z^+ = V + l\phi, \quad Z^- = V - (l+1)\phi. \quad (\text{A.11})$$

In terms of Z^\pm the equations of the fluctuations are:

$$\begin{aligned} \mathcal{O}^2 Z^+ &= \left[l(l+1) \frac{\rho^2}{(q^2 + \rho^4 h^2)C} + \frac{l+1}{\rho} \right] Z^+, \\ \mathcal{O}^2 Z^- &= \left[l(l+1) \frac{\rho^2}{(q^2 + \rho^4 h^2)C} - \frac{l}{\rho} \right] Z^-. \end{aligned} \quad (\text{A.12})$$

Following [21], one can analytically map eqs. (A.12) to the one for the transverse scalars (3.37). First of all, let us introduce the reduced variables ϱ , \bar{M} and \bar{q} , defined as:

$$\varrho = \frac{\rho}{L}, \quad \bar{M}^2 = -\frac{R^{4\alpha}}{L^{4\alpha-2}} k^2, \quad \bar{q} = \frac{q}{L^{2(1-\alpha)} R^{2\alpha}}. \quad (\text{A.13})$$

Then, after substituting $Z^\pm = e^{ikx} \phi^\pm$ in (A.12) and a little algebra, the equation for these modes can be written as:

$$\varrho^{l+1} \partial_\varrho (\varrho^4 Q \partial_\varrho \phi^+) + \left[\bar{M}^2 \varrho^{l+1} - (l+1) \partial_\varrho (\varrho^{4+l} Q) \right] \phi^+ = 0, \quad (\text{A.14})$$

$$\varrho^{-l} \partial_\varrho (\varrho^4 Q \partial_\varrho \phi^-) + \left[\bar{M}^2 \varrho^{-l} + l \partial_\varrho (\varrho^{3-l} Q) \right] \phi^- = 0, \quad (\text{A.15})$$

where $Q = Q(\varrho)$ is the following function:

$$Q(\varrho) \equiv \frac{1}{\bar{q}^2 + \frac{\varrho^4}{(1+\varrho^2)^{2\alpha}}}. \quad (\text{A.16})$$

Moreover, in terms of these reduced variables, the equation for the transverse scalars (3.37) reads:

$$\partial_\varrho (\varrho^2 \partial_\varrho \xi) + \left[\frac{\bar{M}^2}{\varrho^2 Q} - l(l+1) \right] \xi = 0. \quad (\text{A.17})$$

In order to relate (A.17) and (A.14) let us rewrite $\xi = \varrho^{-(l+1)} F^+$ and multiply the transverse scalar equation (A.17) by $\varrho^{l+3} Q$. Then, one can check that the term with F^+ has a constant coefficient and, therefore, once we differentiate with respect to ϱ , the function F^+ appears in the equation only through its derivatives. Then, upon defining $\partial_\varrho F^+ = \varrho^l g^+$, we conclude that the resulting equation for g^+ is simply:

$$\varrho^l \partial_\varrho (\varrho^4 Q \partial_\varrho g^+) + \left[\bar{M}^2 \varrho^l - l \partial_\varrho (\varrho^{3+l} Q) \right] g^+ = 0. \quad (\text{A.18})$$

This equation is exactly the same as the one for the ϕ^+ mode (eq. (A.14)) once we identify $l+1$ in the ϕ^+ equation with l in the equation for g^+ .

It is easy to see that an alternative route can be followed relating the transverse scalar equation (A.17) to the equation (A.15) for ϕ^- , namely by defining $\xi = \varrho^l F^-$. Then, after

multiplying the equation by $\varrho^{2-l} Q$ and taking the ϱ derivative, we see that, again, F^- appears only through its derivatives. Then, we can define $\partial_\varrho F^- = \varrho^{-(l+1)} g^-$ and the equation for g^- becomes:

$$\varrho^{-(l+1)} \partial_\varrho (\varrho^4 Q \partial_\varrho g) + \left[\bar{M}^2 \varrho^{-(l+1)} + (l+1) \partial_\varrho (\varrho^{3-(l+1)} Q) \right] g^- = 0, \quad (\text{A.19})$$

which, indeed, is identical to the equation (A.15) for the ϕ^- modes once we take into account that l is now to be identified with $l+1$ in the equation for g^- .

To sum up, we have that the mapping of [21]:

$$\begin{aligned} \phi_{l=L}^+ &= \varrho^{-L-1} \partial_\varrho (\varrho^{L+2} \xi_{l=L+1}) , \\ \phi_{l=L}^- &= \varrho^L \partial_\varrho (\varrho^{1-L} \xi_{l=L-1}) , \end{aligned} \quad (\text{A.20})$$

also works in the Dp-D(p+2) intersection with flux studied here. As a consequence of this result we can conclude that the mass spectrum of the type I modes displays the same features of that corresponding to the transverse scalars, namely it is continuous and has no mass gap.

A.2 Type II modes

Consider now a configuration with $x = 0$ and take the following ansatz for the gauge field:

$$a_\mu = \phi_\mu(x, \rho) Y^l(S^2), \quad a_\rho = 0, \quad a_i = 0, \quad (\text{A.21})$$

with the extra condition on ϕ :

$$\partial^\mu \phi_\mu = 0. \quad (\text{A.22})$$

Due to this condition, since $x = 0$, the equations of motion for x , a_ρ and a_i are trivially satisfied. The only remaining non-trivial equation is that for a_μ , which reads:

$$\left[\rho^2 h^2 + \frac{q^2}{\rho^2} \right] \partial^\nu \partial_\nu \phi_\mu + \partial_\rho (\rho^2 \partial_\rho \phi_\mu) - l(l+1) \phi_\mu = 0. \quad (\text{A.23})$$

Now, if we write ϕ_μ in a plane-wave basis:

$$\phi_\mu = e^{ikx} \xi_\mu, \quad (\text{A.24})$$

then this equation becomes:

$$\partial_\rho (\rho^2 \partial_\rho \xi_\mu) + \left\{ \left[\rho^2 h^2 + \frac{q^2}{\rho^2} \right] M^2 - l(l+1) \right\} \xi_\mu = 0, \quad (\text{A.25})$$

notice that this equation is the same as that in (3.37) for the transverse scalars.

A.3 Type III modes

Let us take now as ansatz for the gauge field:

$$a_\mu = 0, \quad a_\rho = \phi(x, \rho) Y^l(S^2), \quad a_i = \tilde{\phi}(x, \rho) Y_i^l(S^2). \quad (\text{A.26})$$

With this ansatz it is straightforward to check that $f_{ij} = 0$. Therefore the equation of motion for x is directly satisfied if we take $x = 0$. This leads to an equation of motion of a_i which reads:

$$\partial^\mu \partial_\mu \tilde{\phi} + \partial_\rho \left[\frac{\rho^4}{q^2 + \rho^4 h^2} (\partial_\rho \tilde{\phi} - \phi) \right] = 0. \quad (\text{A.27})$$

Moreover, the equation of motion of a_ρ is:

$$\rho^2 \partial^\mu \partial_\mu \phi + l(l+1) \frac{\rho^4}{q^2 + \rho^4 h^2} (\partial_\rho \tilde{\phi} - \phi) = 0, \quad (\text{A.28})$$

while that of a_μ is:

$$\partial_\mu (l(l+1) \tilde{\phi} - \partial_\rho (\rho^2 \phi)) = 0. \quad (\text{A.29})$$

Clearly, eq. (A.29) is satisfied if:

$$l(l+1) \tilde{\phi} = c + \partial_\rho (\rho^2 \phi), \quad (\text{A.30})$$

where c is an integration constant. Given this condition, it is straightforward to see that the remaining two equations are indeed equivalent. Thus, we arrive to the following equation for ϕ :

$$\rho^2 \partial^\mu \partial_\mu \phi - l(l+1) \frac{\rho^4}{q^2 + \rho^4 h^2} \phi + \frac{\rho^4}{q^2 + \rho^4 h^2} \partial_\rho^2 (\rho^2 \phi) = 0. \quad (\text{A.31})$$

Writing $\phi = e^{ikx} \zeta(\rho)$ with $M^2 = -k^2$, we get the following differential equation for $\zeta(\rho)$:

$$\partial_\rho^2 (\rho^2 \zeta) + \left[\left(h^2 \rho^2 + \frac{q^2}{\rho^2} \right) M^2 - l(l+1) \right] \zeta = 0. \quad (\text{A.32})$$

Eq. (A.32) can also be easily related to the one corresponding to the transverse scalars. Indeed, it is a simple exercise to verify that, if one defines $\xi = \rho \zeta$, eq. (A.32) becomes exactly (3.37). In particular, this fact implies that the mass spectra of these type III modes is also continuous and gapless.

B. Supersymmetry of the M2-M5 intersection

In this appendix we will verify that the M2-M5 intersections with flux studied in section 4 are supersymmetric. We will verify this statement by looking at the kappa symmetry of the M5-brane embedding, which previously requires the knowledge of the Killing spinors of the background. In order to write these spinors in a convenient way, let us rewrite the $AdS_4 \times S^7$ near-horizon metric (4.2) of the M2-brane background as:

$$ds^2 = \frac{r^4}{R^4} dx_{1,2}^2 + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_7^2, \quad (\text{B.1})$$

where $d\Omega_7^2$ is the line element of a unit seven-sphere and R is given in eq. (4.3). In what follows we shall represent the metric of S^7 in terms of polar coordinates $\theta^1, \dots, \theta^7$:

$$d\Omega_7^2 = (d\theta^1)^2 + \sum_{k=2}^7 \left(\prod_{j=1}^{k-1} (\sin \theta^j)^2 \right) (d\theta^k)^2 . \quad (\text{B.2})$$

Moreover, we shall consider the vielbein:

$$\begin{aligned} e^{x^\mu} &= \frac{r^2}{R^2} dx^\mu , & (\mu = 0, 1, 2) , \\ e^r &= \frac{R}{r} dr , \\ e^{\theta^i} &= R \left(\prod_{j=1}^{i-1} \sin \theta^j \right) d\theta^i , & (i = 1, \dots, 7) , \end{aligned} \quad (\text{B.3})$$

where, in the last line, it is understood that for $i = 1$ the product is absent.

The Killing spinors of this background are obtained by solving the equation $\delta\psi_M = 0$, where the supersymmetric variation of the gravitino in eleven dimensional supergravity is given by:

$$\delta\psi_M = D_M \epsilon + \frac{1}{288} \left(\Gamma_M^{N_1 \dots N_4} - 8\delta_M^{N_1} \Gamma^{N_2 \dots N_4} \right) \epsilon F_{N_1 \dots N_4}^{(4)} . \quad (\text{B.4})$$

In eq. (B.4) $F^{(4)} = dC^{(3)}$, where $C^{(3)}$ has been written in eq. (4.4). In order to write equation (B.4) more explicitly, let us define the matrix:

$$\Gamma_* \equiv \Gamma_{x^0 x^1 x^2} . \quad (\text{B.5})$$

Notice that $\Gamma_*^2 = 1$. From the equations $\delta\psi_{x^\alpha} = \delta\psi_r = 0$ we get the value of the derivatives of ϵ with respect to the AdS_4 coordinates, namely:

$$\begin{aligned} \partial_{x^\alpha} \epsilon &= -\frac{r^2}{R^3} \Gamma_{x^\alpha r} (1 + \Gamma_*) , \\ \partial_r \epsilon &= -\frac{1}{r} \Gamma_* \epsilon . \end{aligned} \quad (\text{B.6})$$

First of all, let us solve the first equation in (B.6) by taking $\epsilon = \epsilon_1$ with $\Gamma_* \epsilon_1 = -\epsilon_1$, where ϵ_1 is independent of the Minkowski coordinates x^α . The second equation in (B.6) fixes the dependence of ϵ on r , which is

$$\epsilon_1 = r\eta_1 , \quad \Gamma_* \eta_1 = -\eta_1 , \quad (\text{B.7})$$

where η_1 only depends on the coordinates of the S^7 .

Let us now find a second solution of eq. (B.6), given by the ansatz:

$$\epsilon_2 = (f(r)\Gamma_r + g(r)x^\alpha \Gamma_{x^\alpha}) \eta_2 , \quad (\text{B.8})$$

where $f(r)$ and $g(r)$ are functions to be determined and η_2 is a spinor independent of x^α and r . By plugging this ansatz in (B.6) we get the conditions:

$$\begin{aligned} g(r) &= -\frac{2r^2}{R^3} f(r), & \Gamma_* \eta_2 &= -\eta_2, \\ f'(r) &= -\frac{f(r)}{r}, & g'(r) &= \frac{g(r)}{r}, \end{aligned} \tag{B.9}$$

which can be immediately integrated, giving rise to the following spinor:

$$\epsilon_2 = \left(\frac{1}{r} \Gamma_r - \frac{2r}{R^3} x^\alpha \Gamma_{x^\alpha} \right) \eta_2, \quad \Gamma_* \eta_2 = -\eta_2, \tag{B.10}$$

where $\eta_2 = \eta_2(\theta)$. Then, a general Killing spinor of $AdS_4 \times S^7$ can be written as $\epsilon_1 + \epsilon_2$, namely as:

$$\epsilon = r \eta_1(\theta) + \left(\frac{1}{r} \Gamma_r - \frac{2r}{R^3} x^\alpha \Gamma_{x^\alpha} \right) \eta_2(\theta), \quad \Gamma_* \eta_i(\theta) = -\eta_i(\theta). \tag{B.11}$$

The dependence of the η_i 's on the angle θ^1 can be determined from the condition $\delta\psi_{\theta^1} = 0$, which reduces to:

$$\partial_{\theta^1} \epsilon = -\frac{1}{2} \Gamma_{r\theta^1} \Gamma_* \epsilon. \tag{B.12}$$

It can be checked that eq. (B.12) gives rise to the following dependence of the spinor η_i on the angle θ^1 :

$$\eta_i = e^{\frac{\theta^1}{2} \Gamma_{r\theta^1}} \tilde{\eta}_i, \tag{B.13}$$

where $\tilde{\eta}_i$ does not depend on θ^1 . Similarly, one can get the dependence of the η_i 's on the other angles of the seven-sphere. The result can be written as:

$$\eta_i(\theta) = U(\theta) \hat{\eta}_i, \tag{B.14}$$

with $\hat{\eta}_i$ being constant spinors such that $\Gamma_* \hat{\eta}_i = -\hat{\eta}_i$ and $U(\theta)$ is the following rotation matrix:

$$U(\theta) = e^{\frac{\theta^1}{2} \Gamma_{r\theta^1}} \prod_{j=2}^7 e^{\frac{\theta^j}{2} \Gamma_{\theta^{j-1} \theta^j}}. \tag{B.15}$$

Notice that ϵ depends on two arbitrary constant spinors $\hat{\eta}_1$ and $\hat{\eta}_2$ of sixteen components each one and, thus, this background has the maximal number of supersymmetries, namely thirty-two.

B.1 Kappa symmetry

The number of supersymmetries preserved by the M5-brane probe is the number of independent solutions of the equation $\Gamma_\kappa \epsilon = \epsilon$, where ϵ is one of the Killing spinors (B.11) and Γ_κ is the kappa symmetry matrix of the PST formalism [48, 50]. In order to write the expression of this matrix, let us define the following quantities:

$$\nu_p \equiv \frac{\partial_p a}{\sqrt{-(\partial a)^2}}, \quad t^m \equiv \frac{1}{8} \epsilon^{mn_1 n_2 p_1 p_2 q} \tilde{H}_{n_1 n_2} \tilde{H}_{p_1 p_2} \nu_q. \tag{B.16}$$

Then, the kappa symmetry matrix is:

$$\Gamma_\kappa = -\frac{\nu_m \gamma^m}{\sqrt{-\det(g + \tilde{H})}} \left[\gamma_n t^n + \frac{\sqrt{-g}}{2} \gamma^{np} \tilde{H}_{np} + \frac{1}{5!} \gamma_{i_1 \dots i_5} \epsilon^{i_1 \dots i_5 n} \nu_n \right]. \quad (\text{B.17})$$

In eq. (B.17) g is the induced metric on the worldvolume, $\gamma_{i_1 i_2 \dots}$ are antisymmetrized products of the worldvolume Dirac matrices $\gamma_i = \partial_i X^M E_M^N \Gamma_N$ and the indices are raised with the inverse of g .

We shall consider here the embedding with $L = 0$, which corresponds to having massless quarks. In the polar coordinates we are using for the S^7 this corresponds to taking:

$$\theta^1 = \dots = \theta^4 = \frac{\pi}{2}. \quad (\text{B.18})$$

Moreover, we shall denote the three remaining angles of the S^7 as $\chi^i \equiv \theta^{4+i}$, ($i = 1, 2, 3$). We will describe the M5-brane embeddings by means of the following set of worldvolume coordinates:

$$\xi^i = (x^0, x^1, r, \chi^1, \chi^2, \chi^3), \quad (\text{B.19})$$

and we will assume that:

$$x \equiv x^2 = x(r). \quad (\text{B.20})$$

The induced metric for such embedding is given by (4.12) with $L = 0$ and $\rho = r$, namely:

$$g_{ij} d\xi^i d\xi^j = \frac{r^4}{R^4} dx_{1,1}^2 + \frac{R^2}{r^2} \left(1 + \frac{r^6}{R^6} x'^2 \right) dr^2 + R^2 d\Omega_3^2. \quad (\text{B.21})$$

The induced Dirac matrices for this embedding are:

$$\begin{aligned} \gamma_{x^\mu} &= \frac{r^2}{R^2} \Gamma_{x^\mu}, & (\mu = 0, 1), \\ \gamma_r &= \frac{R}{r} \left(\Gamma_r + \frac{r^3}{R^3} x' \Gamma_{x^2} \right), \\ \gamma_{\Omega_3} &\equiv \gamma_{\chi^1 \chi^2 \chi^3} = R^3 \sqrt{\bar{g}} \Gamma_{\Omega_3}, \end{aligned} \quad (\text{B.22})$$

where $\sqrt{\bar{g}} = \sin^2 \chi^1 \sin \chi^2$ and $\Gamma_{\Omega_3} \equiv \Gamma_{\chi^1 \chi^2 \chi^3}$. Notice also that:

$$\begin{aligned} \gamma^{x^0} &= -\frac{R^2}{r^2} \Gamma_{x^0}, \\ \gamma^{x^1} &= \frac{R^2}{r^2} \Gamma_{x^1}, \\ \gamma^r &= \frac{r}{R} \left(1 + \frac{r^6}{R^6} x'^2 \right)^{-1} \left(\Gamma_r + \frac{r^3}{R^3} x' \Gamma_{x^2} \right). \end{aligned} \quad (\text{B.23})$$

We will also assume that we have switched on a magnetic worldvolume gauge field F , parametrized as in (4.11) in terms of a flux number q . Moreover, in the gauge $a = x^1$, the only non-vanishing component of ν_p is:

$$\nu_{x^1} = -i \frac{r^2}{R^2}, \quad (\text{B.24})$$

and one can check that the only non-vanishing component of \tilde{H} is $\tilde{H}_{x^0 r}$, whose expression is given by (4.14) with $L = 0$ and $\rho = r$. Also, the worldvolume vector t^m defined in (B.16) is zero and one can verify that the kappa symmetry matrix Γ_κ takes the form:

$$\Gamma_\kappa = \frac{\sqrt{\tilde{g}} r^3}{\sqrt{-\det(g + \tilde{H})}} \left(\frac{q}{R^3} + \Gamma_{\Omega_3} \right) \left(\Gamma_{rx^2} + \frac{r^3}{R^3} x' \right) \Gamma_* . \quad (\text{B.25})$$

For the embeddings we are interested in the function $x(r)$ is given by:

$$x = \bar{x} + \frac{q}{2} \frac{1}{r^2} , \quad (\text{B.26})$$

where \bar{x} is a constant (see eq. (4.18)). In order to express the form of Γ_κ for these embeddings, let us define the matrix \mathcal{P} as:

$$\mathcal{P} \equiv \Gamma_{x^2 r} \Gamma_{\Omega_3} . \quad (\text{B.27})$$

Notice that $\mathcal{P}^2 = 1$. Moreover, the kappa symmetry matrix can be written as:

$$\Gamma_\kappa = -\frac{1}{1 + \frac{q^2}{R^6}} \left(\mathcal{P} + \frac{q^2}{R^6} + \frac{q}{R^3} \Gamma_{x^2 r} (1 - \mathcal{P}) \right) \Gamma_* . \quad (\text{B.28})$$

Let us represent the Killing spinors ϵ on the M5-brane worldvolume as is eq. (B.11). By using the explicit function $x(r)$ written in eq. (B.26), one gets:

$$\epsilon = \frac{1}{r} \left(\Gamma_r \eta_2 - \frac{q}{R^3} \Gamma_{x^2} \eta_2 \right) + r \left(\eta_1 - \frac{2\bar{x}}{R^3} \Gamma_{x^2} \eta_2 \right) - \frac{2r}{R^3} x^p \Gamma_{x^p} \eta_2 , \quad (\text{B.29})$$

where the index p can take the values 0, 1 and we have organized the right-hand side of (B.29) according to the different dependences on r and x^p . By substituting (B.28) and (B.29) into the equation $\Gamma_\kappa \epsilon = \epsilon$ and comparing the terms on the two sides of this equation that have the same dependence on the coordinates, one gets the following three equations:

$$\begin{aligned} & (\mathcal{P} + 1) \eta_2 = 0 , \\ & \left[1 - \frac{q}{R^3} \Gamma_{x^2 r} \right] (\mathcal{P} - 1) \left(\eta_1 - \frac{2\bar{x}}{R^3} \Gamma_{x^2} \eta_2 \right) = 0 , \\ & \left[\frac{q}{R^3} \Gamma_{x^2 r} - 1 \right] \Gamma_{x^p} (\mathcal{P} + 1) \eta_2 = 0 . \end{aligned} \quad (\text{B.30})$$

In order to solve these equations, let us classify the sixteen spinors η_1 according to their \mathcal{P} -eigenvalue as:

$$\mathcal{P} \eta_1^{(\pm)} = \pm \eta_1^{(\pm)} . \quad (\text{B.31})$$

Notice that \mathcal{P} and Γ_* commute and, then, the condition of having well-defined \mathcal{P} -eigenvalue is perfectly compatible with having negative Γ_* -chirality. We can now solve the system (B.30) by taking $\eta_2 = 0$ (which solves the first and third equation) and choosing

η_1 to be one of the eight spinors $\eta_1^{(+)}$ of positive \mathcal{P} -eigenvalue. Thus, this solution of (B.30) is:

$$\eta_1 = \eta_1^{(+)} , \quad \eta_2 = 0 . \tag{B.32}$$

Another set of solutions corresponds to taking spinors $\eta_1^{(-)}$ of negative \mathcal{P} -eigenvalue and a spinor η_2 related to $\eta_1^{(-)}$ as follows:

$$\eta_2 = \frac{R^3}{2\bar{x}} \Gamma_{x^2} \eta_1^{(-)} . \tag{B.33}$$

Notice that the second equation in (B.30) is automatically satisfied. Moreover, as $[\mathcal{P}, \Gamma_{x^2}] = 0$, the spinor η_2 in (B.33) has negative \mathcal{P} -eigenvalue and, therefore, the first and third equation in the system (B.30) are also satisfied.

In order to complete the proof of the supersymmetry of our M2-M5 configuration we must verify that the kappa symmetry conditions found above can be fulfilled at all points of the M5-brane worldvolume. Notice that, when evaluated for the embedding (B.18), the spinors $\eta_{1,2}$ depend on the angles χ^i that parametrize the $S^3 \subset S^7$. To ensure that the conditions (B.31) and (B.33) can be imposed at all points of the S^3 we should be able to translate them into some algebraic conditions for the constant spinors $\hat{\eta}_i$. Recall (see eq. (B.14)) that the spinors η_i and $\hat{\eta}_i$ are related by means of the matrix $U(\theta)$. Let us denote by $U_*(\chi)$ the rotation matrix restricted to the worldvolume, i.e.:

$$U_*(\chi) \equiv U(\theta)|_{\theta^1=\dots=\theta^4=\frac{\pi}{2}} . \tag{B.34}$$

Moreover, let us define $\hat{\mathcal{P}}$ as the result of conjugating the matrix \mathcal{P} with the rotation matrix $U_*(\chi)$:

$$\hat{\mathcal{P}} \equiv U_*(\chi)^{-1} \mathcal{P} U_*(\chi) . \tag{B.35}$$

A simple calculation by using (B.15) shows that $\hat{\mathcal{P}}$ is the following constant matrix:

$$\hat{\mathcal{P}} = \Gamma_{x^2\theta^4} \Gamma_{\Omega_3} . \tag{B.36}$$

Moreover, from the definition of $\hat{\mathcal{P}}$ it follows that:

$$\mathcal{P} \eta_1^{(\pm)} = \pm \eta_1^{(\pm)} \iff \hat{\mathcal{P}} \hat{\eta}_1^{(\pm)} = \pm \hat{\eta}_1^{(\pm)} . \tag{B.37}$$

Therefore, the condition (B.31) for η_1 is equivalent to require that the corresponding constant spinor $\hat{\eta}_1$ be an eigenstate of the constant matrix $\hat{\mathcal{P}}$. Finally, as $[U_*, \Gamma_{x^2}] = 0$, eq. (B.33) is equivalent to the following condition, to be satisfied by the constant spinors $\hat{\eta}_1$ and $\hat{\eta}_2$:

$$\hat{\eta}_2 = \frac{R^3}{2\bar{x}} \Gamma_{x^2} \hat{\eta}_1^{(-)} . \tag{B.38}$$

Taken together, these results prove that the kappa symmetry condition $\Gamma_\kappa \epsilon = \epsilon$ can be imposed at all points of the worldvolume of our M5-brane embedding and that this configuration is $\frac{1}{2}$ -supersymmetric.

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